

# Dynamical flows through Dark Matter Haloes II : One and Two points Statistics at the virial radius

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## ABSTRACT

In a serie of three papers, the dynamical interplay between environments and dark matter haloes is investigated, while focussing on the dynamical flows through the virtual virial sphere. It relies on both cosmological simulations, to constrains the environments, and an extension to the classical matrix method to derive the responses of halo. A companion paper (Pichon & Aubert (2006), paper I) showed how perturbation theory allows to propagate the statistical properties of the environment down to a ensemble description of the dynamical response of the embedded halo. The current paper focuses on the statistical characterisation of the environments surrounding haloes, using a set of large scale simulations; the large statistic of environments presented here allow to put quantitative and statistically significant constrains on the properties of flows accreted by haloes.

The description chosen in this paper relies on a “fluid” halocentric representation. The interactions between the halo and its environment are investigated in terms of a time dependent external tidal field and a source term characterizing the infall. The former accounts for fly bys and interlopers. The latter stand for the distribution function of the matter accreted through the virial sphere. The method of separation of variables is used to decouple the temporal evolution of these two quantities from their angular and velocity dependence by means of projection on a 5D basis function for the source.

It is shown how the flux densities of mass, momentum and energy can provide an alternative description to the 5D projection of the source. Such a description is well suited to re-generate synthetic time-lines of accretion which are consistent with environments found in simulations as discussed in appendix. The method leading to the measurements of these quantities in simulations is presented in details and applied to 15000 haloes, with masses between  $5 \cdot 10^{12} M_{\odot}$  and  $10^{14} M_{\odot}$  evolving between  $z = 1$  and  $z = 0$ . The influence of resolution, class of mass, and selection biases are investigated with higher resolution simulations. The emphasis is put on the one and two-point statistics of the tidal field, and of the flux density of mass, while the full characterization of the other fields is postponed to paper III.

The net accretion at the Virial radius is found to decrease with time. This decline results from both an absolute decrease of infall and from a growing contribution of outflows. Infall is found to be mainly radial and occurring at velocities  $\sim 0.75$  times the virial velocity. Outflows are also detected through the virial sphere and occur at lower velocities  $\sim 0.6V_c$  on more circular orbits. The external tidal field is found to be strongly quadrupolar and mostly stationary, possibly reflecting the distribution of matter in the halo's near environment. The coherence time of the small scale fluctuations of the potential hints a possible anisotropic distribution of accreted satellites. The flux density of mass on the virial sphere appears to be more clustered than the potential while the shape of its angular power spectrum seems stationnary. Most of these results are tabulated with simple fitting laws and are found to be consistent with published work which rely on a description of accretion in terms of satellites.

**Key words:** Cosmology: simulations, Galaxies: formation, dynamics.

## 1 GALAXIES IN THEIR ENVIRONMENT

Examples of galaxies interacting with their environments are numerous. The antennae, the cartwheel galaxy, M51 are among the most famous ones. One of our closest neighbor, M31, exhibits a giant stellar stream which may be associated with its satellites (e.g. McConnachie et al. (2003)). Even the Milky Way shows relics of past interactions with material coming from the outskirts, such as the Sagittarius dwarf (Ibata et al. (1995)). It appears clearly that the evolution of galactic systems cannot be understood only by considering their internal properties but also by taking into account their environment. From a dynamical point of view it is still not clear for example if spirals in galaxies are induced by intrinsic unstable modes (e.g. Lynden-Bell & Kalnajs (1972), Kalnajs (1977)) or if they are due to gravitational interactions with satellites or other galaxies (e.g. Toomre & Toomre (1972)). Similarly, normal mode theories of warps have been proposed (Sparke & Casertano (1988), Hunter & Toomre (1969)) but failed to reproduce long-lived warps in a live halo for example (e.g. Binney et al. (1998)). Since warped galaxies are likely to have companions (Reshetnikov & Combes (1998)), it is natural to suggest satellite tidal forcing as a generating mechanism (e.g. Weinberg (1998), Tsuchiya (2002)). Another possibility is angular momentum misalignment of infalling material (e.g. Ostriker & Binney (1989), Jiang & Binney (1999)). The existence of the thick disk may also be explained by past small mergers (e.g. Velazquez & White (1999), Quinn et al. (1993), Walker et al. (1996)). Conversely, very thin disk put serious constraints on the amplitude of the interactions they may have experienced in the past.

On a larger scale, dark matter halos are built in a hierarchical fashion within the CDM model. Some of the most serious challenges that these models are now facing (the over production of dwarf galaxies in the local group (e.g. Moore et al. (1999), Klypin et al. (1999)), the cuspid crisis of NFW-like halos (e.g. Flores & Primack (1994), Moore (1994)), the over-cooling problem and the momentum crisis for galactic disks (e.g. Navarro & Steinmetz (1997)) occur at these scales; it is therefore important to study the effects of the cosmological paradigm on the evolution of galaxies in order to address these issues.

In fact, the properties of galaxies naturally present correlations with their environments. For example Tormen (1997) showed that the shape of halos tends to be aligned with the surrounding satellites distribution. Also, halos spin is sensitive to recently accreted angular momentum (e.g. van Haarlem & van de Weygaert (1993), Aubert et al. (2004)). More generally, halos inherit the properties of their progenitors.

At this point, a question naturally arises, namely ‘what is the dynamical response of a galactic system (halo+disk) to its environment?’. One way to address this issue is to compute high resolution simulations of galaxies into a given environment (e.g. Abadi et al. (2003), Knebe et al. (2004), Gill et al. (2004)). However, if one is interested in reproducing the variety of dynamical responses of galaxies to various environments, the use of such simulations becomes rapidly tedious. An alternative way to investigate this topic is presented here, which should complement both high resolution simulations and large cosmological simulations. In a serie of three papers, an hybrid approach is presented to investigate the interplay between environments and haloes. It relies on both cosmological simulations (to constrains the environments) and on a straightforward extension of the classical tools of galactic dynamics (to derive the haloes response). A companion paper (Pichon & Aubert (2006), in press) describes the analytic theory which allow to assess the dy-

namics of haloes in the open, secular and non-linear régimes. The purpose of the current paper is to set out a framework in which to describe statistically the environments of haloes and present results on the tidal field and the flux density of matter. Paper III (Aubert & Pichon (2006), in prep.) will conclude the complete description of the environments of dark haloes.

### 1.1 Galactic infall as a cosmic boundary

Clearly a number of problems concerning galactic evolution can only be tackled properly via a detailed statistical investigation. Let us briefly make an analogy to the cosmological growth of density fluctuations. Under certain assumptions, one can solve the equations of evolution of those over-densities in an expanding universe (e.g. Peebles (1980), see Bernardeau et al. (2002) for an extensive review). Their statistical evolution due to gravitational clustering follows, given the statistical properties of the initial density field. For example, the power spectrum,  $P(z, k)$ , may be computed for various primordial power spectra,  $P_{\text{prim}}(k)$ , and for various cosmologies. In other words, the statistical properties of the initial conditions are *propagated* to a given redshift through an operator  $\aleph$  given by the non linear dynamical equations of the clustering :

$$P(z, k) = \aleph(P_{\text{prim}}(k), z). \quad (1)$$

In a similar way, how would the statistical properties of environments be propagated to the dynamical properties of galactic systems? This is clearly a daunting task : the previous analogy with the cosmological growth of perturbation is restricted to its principle. For example the assumption of a uniform and cold initial state cannot be sustained for galaxies and halos. While spatial isotropy is clearly not satisfied by disks, and hot, possibly triaxial halos, the velocity tensor of galaxies may also be anisotropic. Environments also share these inhomogeneous and anisotropic features since they are also the product of gravitational clustering and cannot be simply described as Gaussian fields. These boundary conditions are not pure ‘initial conditions’ since they evolve with time and in a non stationary manner (e.g. the accretion rate decreases with time). A whole range of mass must be taken into account, each with different statistical properties. Finally, trajectories cannot be considered as ballistic (even in the linear regime) and must be integrated over long periods. Notwithstanding the above specificities of the galactic framework, two questions have to be answered:

- (i) What is the ‘galactic’ equivalent of  $P_{\text{prim}}(k)$ , *i.e.* how to describe statistically the boundary conditions ?
- (ii) What is the ‘galactic’ equivalent of  $\aleph$ , *i.e.* how to describe the inner galactic dynamics ?

The second point is discussed extensively in Pichon & Aubert (2006) and is briefly summarized in section 2. In that paper, it is shown how a perturbative theory can describe the dynamics of haloes which experience both accretion and tidal interactions (see also Aubert et al. (2004)). Within this formalism, the environment is described by the external gravitational potential and a *source function*. The former describes fly-bys and the tidal field of neighbouring large scale structures. The latter describes the flows of dark matter, *i.e.* the exchanges of material between the halo and the ‘inter-halo’ medium. The knowledge of these two quantities fully characterizes the boundary condition. The focus here is on well formed halos which do not undergo major merger between  $z = 1$  and  $z = 0$ . This bias is consistent with a galactocentric description in which a perturbative description of the inner dynamics is appropriate and we explicitly ignore equal mass mergers. As

briefly explained in section 2, this formalism provides a link between the statistical properties of environments to the statistical distributions of the responses of haloes: this link is referred to as *statistical propagation*. In this manner, the distribution of haloes dynamical state can be directly inferred from the statistical properties of environments, without relying on the follow up of individual interacting haloes. The observed distributions of dynamical features provides informations on the cosmic boundaries which influence haloes. This, together with the perturbative formalism described in Pichon & Aubert (2006) should allow us to address statistically the recurrent ‘nurture or nature’ problem of structure formation within galactic systems. All processes which depend critically on the geometry of the interactions may be tackled in this framework<sup>1</sup>.

The statistical propagation relies on the knowledge of the properties of the environment and is stated by the point (i) mentioned above. This question is investigated the current paper by using a large set of simulations, where each halo provides a realization of the environment. From this large ensemble of interacting haloes, the aim is to extract the global properties of their “cosmic neighborhood”. Such a task requires an appropriate description of the source and the surrounding tidal field. It is the purpose of this work to implement this description which should both provide insights on the generic properties of cosmic environments and be useful in a ‘dynamical’ context. Specifically, a method is presented to constrain the exchanges between the halo and its neighborhood, via the properties of accretion and potential measured on *the virial sphere*. The advantages, specificities and caveats (and the methods implemented to overcome them) provided by this *halocentric* approach will be presented in this paper.

As shown in the following sections, the source function is given by the phase space distribution function (DF hereafter) of the advected material. As a consequence, its full characterization is a complex task since it involves sampling a 5 dimensional space. The characterization of the source is based upon the projection of its DF on a suitable 5D basis. In particular, it is shown how such a description can be used to constrain the kinematic properties of accretion by dark matter haloes in cosmological simulations. The detailed statistical characterization of the higher moments of the source is postponed to a paper III. An alternative description of the source is also presented; it relies on *flux densities* through the virial sphere, i.e. the moments of the source DF. Even though it is less suited to the dynamical propagation, this alternative description is easier to achieve numerically and to interpret physically. In particular, it illustrates how the source term may be characterized statistically via its moments. The link between these flux densities and the 5D projection of the source is discussed together with the one and two-point statistics of the flux densities of mass through haloes in simulations. The mean of reprojecting the effect of external potential inside the halo (through Gauss’s theorem) while knowing its properties on the virial sphere is discussed. The potential’s one and two points statistics are also investigated around simulated haloes and interpreted.

Finally (Section F) provide means of regenerating such flows ab initio from its tabulated statistical properties. Such tools yields a way to embed idealized simulations of galaxies into realistic cosmological environments.

<sup>1</sup> However, all departure to angular isotropy on the sphere will be ignored here (in contrast to what was stressed in Aubert et al. (2004) ), and its implications will be postponed to the discussions in section 8.

The outline of the paper is the following: Section 2 presents briefly the dynamics of open collisionless systems and states the principle of statistical propagation. Section 3 presents the procedure used to compute the source term, and illustrates its implementation on a given halo. The simulations and the corresponding selection biases of our sample are then described in section 4.2. Section 5 and 6 present the statistical measurements for one and two point statistics respectively. Section ?? draws a global picture of galactic infall on  $L^*$  galaxies, while a discussion and conclusions follow in section 8.

## 2 DYNAMICS OF OPEN COLLISION-LESS SYSTEMS

The exchanges occurring between a halo and its environment can be characterized in several ways. One of the classical method involves building a merger tree where the whole history of formation of a halo is expressed in terms of global properties of its progenitors (e.g. Lacey & Cole (1993), Kauffmann & White (1993), Somerville & Kolatt (1999)). While well suited to study the evolution of those characteristics, it cannot be directly applied to predict in details the halos’ inner dynamic because of the lack of spatial information on these interactions. One could track the whole (six dimensional) phase-space history of all the progenitors, but not only would it be difficult to store in practice it would also not give information on the influence of large scale structures through their gravitational potential. In the present paper, following Aubert et al. (2004), it is suggested to measure the relevant quantities on a surface *at the interface* between the halo and the intergalactic medium. Accretion is described as a flux of particles through the halos’ external boundaries.

This section presents an extension of the formalism developed by e.g. Tremaine & Weinberg (1984) and Murali (1999) to open spherical collisionless systems. The dynamics of a dark matter spherical halo is obtained by solving the collisionless Boltzmann equation coupled with the Poisson equation :

$$\partial_t F + \mathbf{v} \cdot \partial_{\mathbf{r}} F - \nabla \Psi \cdot \partial_{\mathbf{v}} F = 0, \quad (2)$$

$$\Delta \Psi = 4\pi G \int d^3 v F(\mathbf{v}). \quad (3)$$

where  $F(\mathbf{r}, \mathbf{v}, t)$  is the system’s distribution function coupled to  $\Psi(\mathbf{r}, t) \equiv \psi + \psi_e$ , the total gravitational potential (self-gravitating + external perturbation). Note that, in a somewhat unconventional manner,  $\psi^e$  refers here to the external potential, i.e. the tidal potential created by the perturbations *outside* the boundary. The gravitational field of incoming particles is accounted for by the source term. Equation (2) coupled with Hamilton’s equations is a conservation equation:

$$\partial_t F + \tilde{\nabla}(\mathbf{u}F) = 0, \quad (4)$$

where  $\mathbf{u} \equiv (\mathbf{v}, -\nabla \Psi)$  and  $\tilde{\nabla} \equiv (\partial_{\mathbf{r}}, \partial_{\mathbf{v}})$ . As a consequence, considering a ‘source of material’ described by  $f_e(\mathbf{w})$  located on a surface  $S(\mathbf{w})$  implies:

$$\partial_t F + \tilde{\nabla}(\mathbf{u}F) = -\delta_D(S(\mathbf{w}) - a)\mathbf{u} \cdot \frac{\nabla S}{|\nabla S|} f_e(\mathbf{w}), \quad (5)$$

where  $\mathbf{w} \equiv (\mathbf{r}, \mathbf{v})$  describes the phase space and  $a$  defines the surface boundary of the studied system (here  $\delta_D$  stands for the Dirac delta function). If this boundary is defined as a spherical surface with radius  $R$ , then equation (5) becomes (e.g. Appel (2002)) :

$$\partial_t F + \tilde{\nabla}(\mathbf{u}F) = -\delta_D(r - R)v_r f_e(\mathbf{w}) \quad (6)$$

$$\equiv -\delta_{\text{D}}(r - R)s^e(\mathbf{w}). \quad (7)$$

The function  $s^e$  will be hereafter referred to as the ‘source’ function. Formally the r.h.s. of equation (7) can be seen as an additional local rate of change of the system’s distribution function. Note that equation (7) involves the external potential,  $\psi^e$ , via  $\mathbf{u}$ .

### 2.1 Moments of the source term

Integrating equation (7) over velocities leads to the mass conservation relation:

$$\partial_t \rho + \nabla \cdot (\mathbf{v}\rho) = -\delta_{\text{D}}(r - R)(\rho v_r)_e \equiv -\delta_{\text{D}}(r - R)\varpi_\rho, \quad (8)$$

where the source appears as an external flux density of matter  $(\rho \bar{v}_r)_e$  or  $\varpi_\rho$ . Taking the next moment of equation (7) leads to the Euler-Jeans Equation:

$$\partial_t \rho \mathbf{v} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \rho \nabla \Psi = -\delta_{\text{D}}(r - R)(\rho v_r \mathbf{v})_e, \quad (9)$$

where the source adds a flux density of momentum,  $\varpi_{\rho \mathbf{v}}$ , to the conventional Jeans equation. Taking the successive moments of equation (7) will generically include a new term in the resulting equations.

### 2.2 Propagating the dynamics

Following Tremaine & Weinberg (1984), equation (7) can be solved along with Poisson’s equation in the regime of small perturbations. In the spirit of paper I let us define the system’s environment by the external perturbative potential  $\psi^e(\mathbf{r}, t)$ , and the source  $s^e(\mathbf{r}, \mathbf{v}, t)$ . Given this environment, the system’s linear potential response  $\psi(\mathbf{r}, t)$  can be computed. Writing the following expansions:

$$\psi(\mathbf{r}, t) = \sum_{\mathbf{n}} a_{\mathbf{n}}(t) \psi^{[\mathbf{n}]}(\mathbf{r}), \quad (10)$$

$$\psi^e(\mathbf{r}, t) = \sum_{\mathbf{n}} b_{\mathbf{n}}(t) \psi^{[\mathbf{n}]}(\mathbf{r}), \quad (11)$$

$$s^e(\mathbf{r}, \mathbf{v}, t) = \sum_{\mathbf{n}} c_{\mathbf{n}}(t) \phi^{[\mathbf{n}]}(\mathbf{r}, \mathbf{v}), \quad (12)$$

where  $\phi^{[\mathbf{n}]}(\mathbf{r}, \mathbf{v})$  and  $\psi^{[\mathbf{n}]}(\mathbf{r})$  are suitable basis functions, and solving the Boltzmann and Poisson’s equation for  $a_{\mathbf{n}}$ , one finds (Aubert et al. (2004)):

$$\mathbf{a}(t) = \int_{-\infty}^{\infty} d\tau \mathbf{K}(\tau - t) \cdot [\mathbf{a}(\tau) + \mathbf{b}(\tau)] + \mathbf{H}(\tau - t) \cdot \mathbf{c}(\tau). \quad (13)$$

The kernels  $\mathbf{K}$  and  $\mathbf{H}$  are functions of the equilibrium state distribution function,  $F_0$ , and of the two basis,  $\phi^{[\mathbf{n}]}(\mathbf{r}, \mathbf{v})$ , and  $\psi^{[\mathbf{n}]}(\mathbf{r})$  only (see paper I). As a consequence, they may be computed once and for all for a given equilibrium model. Since the basis function,  $\psi^{[\mathbf{n}]}$ , can be customized to the NFW-like profile of dark matter halos, it solves consistently and efficiently the coupled dynamical and field equations so long as the entering fluxes of dark matter amounts to a small perturbation in mass compared to the underlying equilibrium.

Assuming linearity and knowledge of  $\mathbf{K}$  and  $\mathbf{H}$ , one can see that the properties of the environments (through  $\mathbf{b}$  and  $\mathbf{c}$ ) are *propagated* exactly to the inner dynamical properties of collisionless systems. Note in particular that the whole phase space response of the halo follow from the knowledge of  $\mathbf{a}$ . For example, taking the temporal Fourier transform of equation (13), the cross correlation

matrix is easily deduced:

$$\langle \hat{\mathbf{a}} \cdot \hat{\mathbf{a}}^{*\top} \rangle = \langle (\mathbf{I} - \hat{\mathbf{K}})^{-1} \cdot [\hat{\mathbf{K}} \cdot \hat{\mathbf{b}} + \hat{\mathbf{H}} \cdot \hat{\mathbf{c}}] \cdot [\hat{\mathbf{K}} \cdot \hat{\mathbf{b}} + \hat{\mathbf{H}} \cdot \hat{\mathbf{c}}]^{*\top} \cdot (\mathbf{I} - \hat{\mathbf{K}})^{-1* \top} \rangle, \quad (14)$$

where  $\hat{\mathbf{x}} = \hat{\mathbf{x}}(\omega)$  is the Fourier Transform of  $\mathbf{x}(t)$ . The environment’s two-points statistic, via  $\langle \hat{\mathbf{b}} \cdot \hat{\mathbf{b}}^{*\top} \rangle$ ,  $\langle \hat{\mathbf{c}} \cdot \hat{\mathbf{c}}^{*\top} \rangle$  and  $\langle \hat{\mathbf{b}} \cdot \hat{\mathbf{c}}^{*\top} \rangle$ , modifies the correlation of the response of the inner halo.

Linear dynamics do not take in account the effects on the perturbation induced by dynamical friction. More generally the damping of incoming fluxes will ultimately require non linear dynamics (since the relative temporal phases of the infall do matter in that context). It is also assumed in equation (13) that the incoming material does not modify the equilibrium state of the system. The secular evolution of the system should also be ultimately taken in account, through a quasi-linear theory for example (see e.g. paper I).

Let us emphasize that, since the addressed problem is linear, the response, equation (13), can be recast into a formulation which only involve an ‘external potential’, namely the sum of  $\psi^e$  and the potential created by the entering particles described by  $s^e$ . While formally simpler at the linear deterministic level, this alternative formulation does not translate well non linearly or statistically (since it would require the full knowledge of the perturbation everywhere in space in a manner which is dependent upon the inner structure of the halo).

In the following sections, our aim is to describe how the two fields  $\psi^e(\mathbf{r}, t)$  and  $s^e(\mathbf{r}, \mathbf{v}, t)$  can be extracted from haloes in cosmological simulations. Then it will be shown how to characterize their statistical properties as a function of time via their expansion coefficients,  $b_{\mathbf{n}}(t)$  and  $c_{\mathbf{n}}(t)$ .

### 2.3 Convention and notations

In what follows, let us characterize the properties of 2 fields, either angularly, kinematically, statistically or temporally, or any combination (for various classes of masses). For a given field,  $X$ , let us introduce the following notations for clarity:

$$\bar{X} \equiv \frac{1}{4\pi} \int X(\theta, \phi) d\sin(\theta) d\phi, \quad (15)$$

which represents the angular average of  $X$  over the sphere. Alternatively, let us define the temporal average over  $\Delta t$  as:

$$\underline{X} \equiv \frac{1}{\Delta T} \int_0^T X(t) dt. \quad (16)$$

Finally let us define the ensemble average as

$$\langle X \rangle \equiv \int X \mathcal{F}(X) dX = E\{X\}, \quad (17)$$

where  $\mathcal{F}$  is the density probability distribution of  $X$ . Here  $E\{X\}$  stands for the expectation of  $X$ . In practice, in section 4.2, an estimator for ensemble average of  $X$  measured for  $N$  halos is given:<sup>2</sup>

$$\langle X \rangle_N = \frac{1}{N} \sum_i^N X_i. \quad (18)$$

<sup>2</sup> An alternative would be to weight the sum by the relative number of objects in each halo, hereby down weighting light halos. It is found that this alternative estimator did not significantly affect our measurements



The underlying probability distributions are sometimes very skewed (when, *e.g.* corresponding to strong or weak accretion event around massive or smaller halos), which requires special care when attempting to define statistical trends. Hence let us also define  $\langle\langle X \rangle\rangle$  as the *mode* (or most probable value) of the fitting distribution of  $F$ .

All external quantities, (flux densities, potential etc) will generally be labeled as  $X^e$ . Let us introduce moments of the source over velocities, which correspond to flux densities, noted  $\varpi_X$  and their corresponding fluxes, noted  $\Phi_X$ . Table (3.3) gives a list of such flux densities, flux pairs. Finally the harmonic transform of the field,  $X$ , will be written as  $a_{\ell,m}^X$  and its corresponding power spectrum  $C_\ell^X$ , while the parameters relative to fitting the statistics of the field will be written as  $q_X$ . Note that the contrast of the field,  $X$  was also introduced as

$$\delta_X \equiv \frac{X - \bar{X}}{\langle \bar{X} \rangle}, \quad (19)$$

and its corresponding harmonic transform,  $\tilde{a}_{\ell m}^X$ .

### 3 THE SOURCE OF INFALL

Let us first describe our strategy to fully characterize the source of cosmic infall at the virial radius via collisionless dark matter simulations, and enumerate the corresponding biases. In particular let us illustrate our procedure on a template halo.

#### 3.1 Describing the source

As argued in section 2, computing the response of an open system to infalling material requires the knowledge of the source function,  $s^e(\mathbf{r}, \mathbf{v}, t)$ . Given the particles accreted by a halo, one possibility involves storing those phase-space properties for all particles. While feasible for a limited number of halos, this task would become rapidly intractable for our large number of simulations. In order to compress the information, the accreted distribution function is projected here on a basis of function, following equation (12).

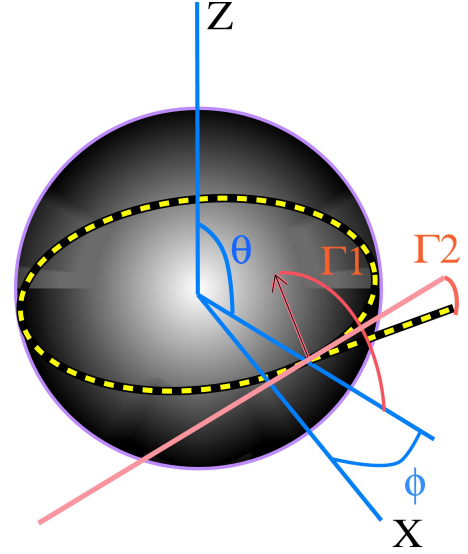
Since the measurement is carried at a fixed radius, the phase space is reduced from six to five degrees of freedom: two for the angular position on the sphere, described by two angles,  $(\theta, \phi) \equiv \boldsymbol{\Omega}$ , and three for the velocity space described in spherical coordinates by  $(v, \Gamma_1, \Gamma_2) = (v, \boldsymbol{\Gamma})$ , where  $v$  is the velocity modulus and  $\boldsymbol{\Gamma}$  are the two angles describing its orientation (see figure 1). The angle  $\Gamma_1$  indicates how radial is the velocity, with  $\Gamma_1 > \pi/2$  for infalling dark matter and  $\Gamma_1 < \pi/2$  for outflows.  $\Gamma_2$  indicates the orientation of the infall's tangential motion.

Recall that the two fields,  $c_n$  (hence  $s^e(\boldsymbol{\Omega}, \boldsymbol{\Gamma}, v, t)$ ) and  $b_n$  (hence  $\psi^e(\boldsymbol{\Omega}, t)$ ) are respectively five dimensional and two dimensional (as a function of mass and time). Note also that both  $s^e$  and  $\psi^e$  are statistically stationary with respect to  $\boldsymbol{\Omega}$ , while  $s^e$  is partially isotropic and not stationary with respect to  $\boldsymbol{\Gamma}$ ; neither  $\psi^e$  and  $s^e$  are stationary with respect to cosmic time.

##### 3.1.1 Harmonic expansion of the incoming fluxes

The  $\boldsymbol{\Omega}$  and  $\boldsymbol{\Gamma}$  dependence are naturally projected on a basis of spherical harmonics,  $Y_{\ell,m}(\boldsymbol{\Omega})$  and  $Y_{\ell',m'}(\boldsymbol{\Gamma})$ . The velocity amplitude dependence is projected on a basis of Gaussian functions,  $g_\alpha(v)$  with means  $\mu_\alpha$  and a given r.m.s.  $\sigma$ . One can write:

$$\phi^{[n]}(\mathbf{r}, \mathbf{v}) = Y_{\mathbf{m}}(\boldsymbol{\Omega}) Y_{\mathbf{m}'}(\boldsymbol{\Gamma}) g_\alpha(v), \quad (20)$$



**Figure 1.** The angles  $\boldsymbol{\Omega}$  and  $\boldsymbol{\Gamma}$ . The dot indicates the position of the particle on the sphere. The dashed ellipse represents the plane which contains both the particle and the sphere center.  $X$  and  $Z$  are arbitrary directions defined by the simulation's box.  $(\theta, \phi) = \boldsymbol{\Omega}$  are the particle's angular coordinates on the sphere.  $\boldsymbol{\Gamma} = (\Gamma_1, \Gamma_2)$  define the orientation of the particle's velocity vector (shown as an arrow).

where  $\mathbf{n} \equiv (\ell, m, \alpha, \ell', m') = (\mathbf{m}, \alpha, \mathbf{m}')$ . The expansion coefficients,  $c_{\mathbf{m}'}^{\mathbf{m}}(t)$  are given by :

$$c_{\mathbf{m}'}^{\mathbf{m}}(t) = (\mathbf{G}^{-1} \cdot \mathbf{s}_{\mathbf{m}'}^{\mathbf{m}})_\alpha. \quad (21)$$

where:

$$(\mathbf{s}_{\mathbf{m}'}^{\mathbf{m}})_\beta = \int d\boldsymbol{\Omega} d\boldsymbol{\Gamma} dv v^2 g_\beta(v) Y_{\mathbf{m}}^*(\boldsymbol{\Omega}) Y_{\mathbf{m}'}(\boldsymbol{\Gamma}) s^e(v, \boldsymbol{\Omega}, \boldsymbol{\Gamma}, t), \quad (22)$$

given

$$\mathbf{G}_{\alpha,\beta} = \int dv v^2 g_\alpha(v) g_\beta(v). \quad (23)$$

Note that the expansion defined in equation (12) where the coefficients are given by equation (21) involves 5 subscripts spanning the 5 dimensional phase space, while the expansion in equation (10) only involve 3 subscripts. This description of the source term is reduced to a set of coefficients which depends on time only. Furthermore this procedure requires to parse the particles only once, and all the momenta (*e.g.* mass flux density, PDF of impact parameter ...) of the source terms can be computed directly from these coefficients. As a consequence, the statistics of momenta follow linearly from the *statistics* of coefficients only, as shown in section 4.2.

##### 3.1.2 Harmonic expansion of the external potential

Let us call  $b'_{\ell m}(t)$  the harmonic coefficients of the expansion of the external potential on the virial sphere. Following Murali (1999), let us expand the potential over the biorthogonal basis,  $(u_n^{\ell m}, d_n^{\ell m})$ , so that

$$\begin{aligned} \psi^e(r, \boldsymbol{\Omega}, t) &= \sum_{n,\ell,m} b'_{\ell m}(t) Y_\ell^m(\boldsymbol{\Omega}) \left(\frac{r}{R_{200}}\right)^\ell, \\ &= \sum_{\mathbf{n}} b_{\mathbf{n}}(t) \psi^{[n]}(\mathbf{r}), \end{aligned} \quad (24)$$

where  $\psi^{[n]}(\mathbf{r}) \equiv Y_\ell^m(\boldsymbol{\Omega})u_j^{\ell m}(r)$ . The first equality in equation (24) corresponds to the inner solution of the three dimensional potential whose boundary condition is given by  $Y_m^\ell(\boldsymbol{\Omega})b'_{\ell m}$  on the sphere of radius  $R_{200}$  (defined below). Since the basis is biorthogonal, it follows that

$$b_n(t) = \left( \int d_n^{\ell m}(r) \left( \frac{r}{R_{200}} \right)^\ell dr \right) b'_{\ell m}(t). \quad (25)$$

It is therefore straightforward to recover the coefficient of the 3D external potential from that of the potential on the sphere.

### 3.2 From simulations to expansion coefficients

Once a halo is detected, its outer ‘boundary’ is defined as a sphere centered on its center of mass with a radius,  $R_{200}$  (or *Virial radius*), defined implicitly by  $3M/(4\pi R_{200}^3) = 200\rho_0$ . This choice of radius is the result of a compromise between being a large distance to the halo center, to limit the contribution of halo’s inner material to fluxes, and being still close enough to the halo’s border, to limit the simulation’s fraction to be processed and avoid contributions of fly by objects.

The time evolution of accretion is measured backwards in time by following the biggest progenitor of each halo detected at redshift  $z = 0$ . The positions and velocities of particles passing through the virial sphere between snapshots are then stored. All positions are measured relative to the biggest progenitor center of mass while velocities are measured relative to its average velocity, for each redshift  $z$ . In one of the simulation described below, the total comoving drift distance of the center of mass was compared to the distance between the halo’s position between  $z=1$  and  $z=0$ . The haloes were chosen to satisfy the criteria described in 4.2. It is found that the scattering of the motion of the center of mass represents less than 10 % of the distance covered in 8 Gyrs. The center of mass of the biggest progenitor seems stable enough to be a reference.

Sticking to the previous definition of  $R_{200}$  would imply a changing outer boundary and an ‘inertial’ flux through a moving surface would have to be taken in account. To overcome this effect, the sphere was kept *constant in time* at a radius equals to  $R_{200}(z = 0)$ . This choice corresponds to a reasonable approximation since the actual virial radius does not change significantly with time between  $z < 2$  for a reasonably smooth accretion history. As shown in figure 2 the virial radius at  $z = 1$  is only 20% smaller than  $R_{200}$  measured at  $z = 0$ . Larger haloes have larger variations but the median value of the difference between the two radii remains smaller than 30% for final masses smaller than  $10^{14} M_\odot$ . Finally, measurements were done using physical coordinates (and not comoving coordinates). These choices were partly guided by the fact that they simplify future applications of these results to the inner dynamic of the halos (see paper I).

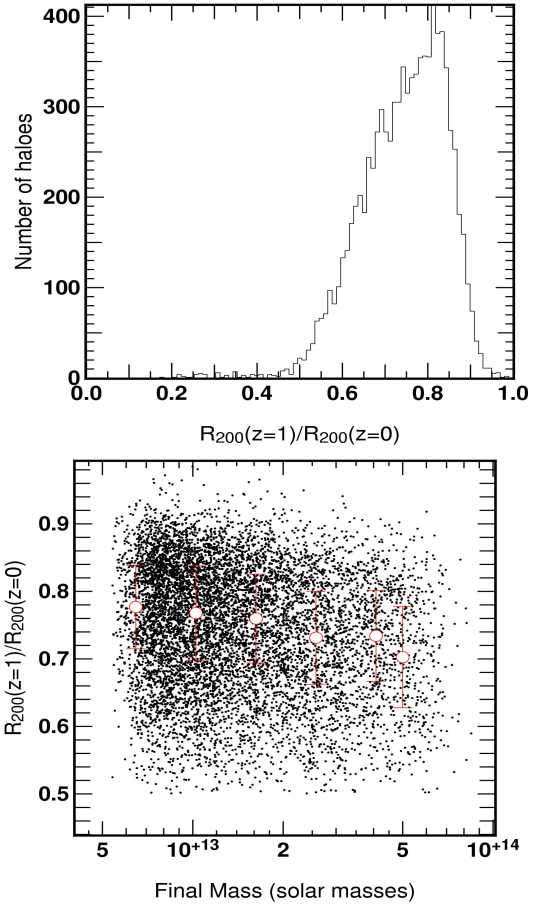
#### 3.2.1 Sampling on the sphere

As shown in section 2, the source function  $s^e$  reads

$$s^e \equiv f(\mathbf{r}, \mathbf{v}, t)v_r = \sum_i \delta_D^3(\mathbf{r} - \mathbf{r}_i(t))\delta_D^3(\mathbf{v} - \mathbf{v}_i(t))v_{r,i}. \quad (26)$$

Switching to spherical coordinates leads to:

$$s^e = \sum_i^N \frac{\delta_D(R_{200} - r_i(t))}{R_{200}^2} \frac{\delta_D(v - v_i(t))}{v^2} \times \quad (27)$$



**Figure 2.** *Top:* the distribution of the ratio between the virial radius measured at  $z = 1$  and at  $z = 0$ . *Bottom:* the distribution of  $R_{200}(z = 1)/R_{200}(z = 0)$  as a function of the halo’s final mass. Each point represent one halo. Symbols stand for the median value of  $R_{200}(z = 1)/R_{200}(z = 0)$  in 6 different classes of masses. Bars stand for the interquartile. The two measurements were performed on 9023 haloes which satisfy the selection criteria defined in section 4.2.

$$\frac{\delta_D(\boldsymbol{\Omega} - \boldsymbol{\Omega}_i(t))}{\sin \Omega_1} \frac{\delta_D(\boldsymbol{\Gamma} - \boldsymbol{\Gamma}_i(t))}{\sin(\Gamma_1)} v_{r,i}(t), \quad (28)$$

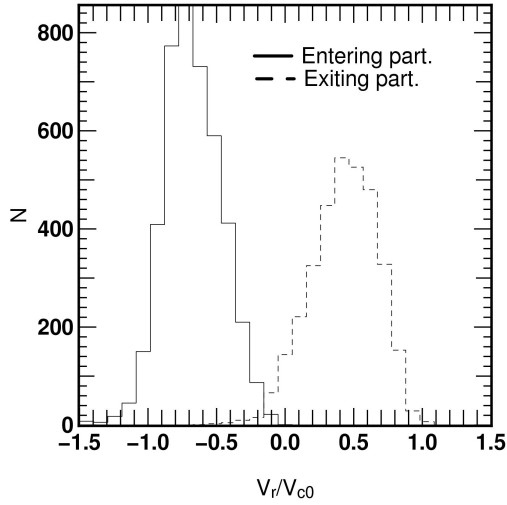
where  $i$  is the particle index. Now:

$$\begin{aligned} v_{r,i} \delta_D(R_{200} - r_i(t)) &= \sum_k v_{r,k,i} \left| \frac{dt}{dr} \right| \delta_D(t - t_{200,k,i}), \\ &= \sum_k w_{k,i} \delta_D(t - t_{200,k,i}), \end{aligned} \quad (29)$$

where  $t_{200,k,i}$  corresponds to the  $k$ -th passage of the  $i$ -th particle through the virtual boundary  $R_{200}$  (and  $v_{r,k,i}$  the corresponding radial velocity). In our conventions, the weight function  $w_{k,i}$  takes the value 1 if the particle is entering and -1 if its exiting the virial sphere. Given that our time resolution is finite, let us consider a time interval  $\Delta T$  around  $t$  and define the (temporal) average phase space flux density over  $\Delta T$ :

$$\underline{s}^e(t) \equiv \frac{1}{\Delta T} \int_t^{t+\Delta T} d\tau s^e(\tau). \quad (30)$$

Equation (28) becomes:



**Figure 3.** The distribution of interpolated radial velocities  $v_r$  of particles passing through the virial radius. Those particles were taken from the whole history of accretion of a typical halo ( $R_{200} = 860$  kpc,  $M(z = 0) = 3 \cdot 10^{13} M_{\odot}$ ). Entering particles (solid line)  $v_r < 0$  while exiting particles (dashed line) have  $v_r > 0$ , as it should be.

$$\underline{s}^e(t) = \sum_{i,k}^N \frac{\delta(v - v_{i,k})}{v^2 \Delta T \cdot R_{200}^2} \frac{\delta(\mathbf{\Omega} - \mathbf{\Omega}_{i,k})}{\sin \Omega_1} \frac{\delta(\mathbf{\Gamma} - \mathbf{\Gamma}_{i,k})}{\sin(\Gamma_1)} w_{i,k}. \quad (31)$$

The simulations were sampled in time regularly in  $\ln(z)$  (i.e.  $\Delta \ln(z) = \text{constant}$ ). From  $z = 2$  to  $z = 0.1$ , 23 snapshots were taken (and a  $z = 0$  snapshot was added to the sample). If  $\Delta t$  is small, the sum over  $k$  should mostly involve one passage, i.e. :

$$\underline{s}^e(t) \sim \frac{1}{\Delta T \cdot R_{200}^2} \sum_i^N \frac{\delta(v - v_i)}{v^2} \frac{\delta(\mathbf{\Omega} - \mathbf{\Omega}_i)}{\sin \Omega_1} \frac{\delta(\mathbf{\Gamma} - \mathbf{\Gamma}_i)}{\sin(\Gamma_1)} w_i. \quad (32)$$

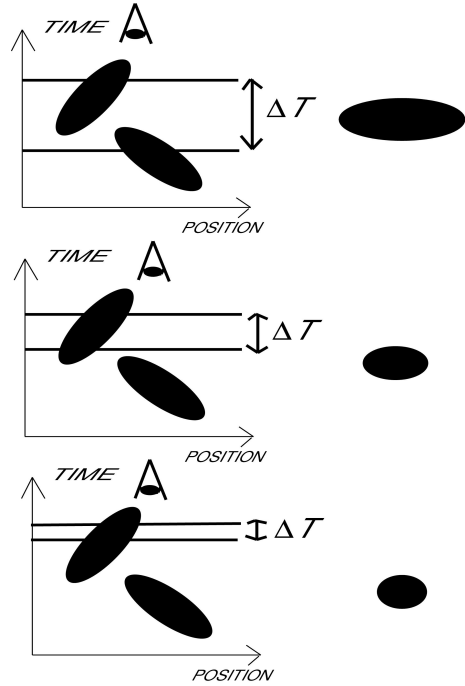
Now these measurements only give access to  $(v, \mathbf{\Omega}, \mathbf{\Gamma})$  at fixed redshift,  $z$ , every varying  $\Delta z$ . Consequently, these values need to be interpolated at the sought  $t_{200,i}$  approximated by :

$$t_{200,i} = t_i(z_n) + \frac{t(z_{n+1}) - t(z_n)}{r_i(z_{n+1}) - r_i(z_n)} (R_{200} - r_i(z_n)). \quad (33)$$

Given these ‘crossing’ instant, the positions,  $\mathbf{r}$ , and velocities,  $\mathbf{v}$ , are also linearly interpolated. For instance, one gets for the  $x$  component of the velocity :

$$v_{x,i}(t_{200}) = v_{x,i}(z_n) + \frac{v_{x,i}(z_{n+1}) - v_{x,i}(z_n)}{t(z_{n+1}) - t(z_n)} (t_{200} - t(z_n)). \quad (34)$$

Such an interpolation is not strictly self-consistent since a ballistic motion requires a constant velocity along the trajectory. The worst-case scenario would correspond to particles which have entered the virial sphere with an outflowing velocity vector and vice-versa. As a simple but important check, the distribution of interpolated radial velocities was plotted (see Fig. (3)). Those were computed from the whole history of accretion of a typical halo ( $R_{200} = 860$  kpc,  $M_{z=0} = 3 \cdot 10^{13} M_{\odot}$ ). The two types of particles (entering/exiting) are confined in their radial velocity plane: entering (resp. exiting) particles have negative (resp. positive) radial velocities. Velocities are correctly interpolated. It also means that our time steps are small enough to ensure a small variation of positions/velocities of particles, validating a posteriori our assumptions. A fraction of exiting particles do have a negative radial velocity but represent less



**Figure 4.** The impact of time averaging on the measured scales of dark matter passing through the sphere. On the left, time-position diagram of dark matter (black ellipses) as they pass through the sphere. Time integration is performed during  $\Delta T$  (the two horizontal lines). On the right the accreted dark matter as seen on the sphere. A longer integration time increases the length scale of the incoming blob. If  $\Delta T$  gets very large, different blobs may be seen as one (upper diagram).

than a few percent of the total population. For safety, those particles are rejected from the following analysis.

One should note that the measured angular scales are sensitive to the time sampling (see Fig. (4)). Increasing the sampling time tends to increase the apparent size of objects as measured on the sphere. Since this increase depend on the shape or the orientation of the objects, this effect *cannot* be simply time-averaged. As a consequence, a varying time step would induce a variation of typical spatial scale. The interpolation given by equation (33) allows also for a constant time step resampling of the source term  $\underline{s}^e$ . All reference to the time average shall be dropped from now on.

Given equation (31), computing the expansions coefficients of  $s_e$  is straightforward :

$$c_{\alpha,m'}^m = \left( \mathbf{G}^{-1} \sum_i^N w_i \frac{\mathbf{g}(v_i) Y_m^*(\mathbf{\Omega}_i) Y_{m'}^*(\mathbf{\Gamma}_i)}{\Delta T \cdot R_{200}^2} \right)_{\alpha}. \quad (35)$$

It is expected that the above procedure is more accurate than the strategy presented in Aubert et al. (2004), where the flux densities were smoothed over a shell of finite thickness ( $R_{200}/10$ ).

The harmonic expansion  $b'_m$  of the external potential  $\psi^e(R_{200}, \mathbf{\Omega})$  is computed directly from the positions of external particles (e.g. Murali & Tremaine (1998)) :

$$b'_{\ell,m}(t) = -\frac{4\pi G}{2\ell + 1} \sum_j^N Y_{\ell m}^*(\mathbf{\Omega}_j(t)) \frac{R_{200}^{\ell}}{r_j^{\ell+1}(t)}, \quad (36)$$

where  $r_j$  and  $\mathbf{\Omega}_j$  are the distance and the two angles defining the position of the  $j$ -th external particles. The quantities  $r_j(t)$  and  $\mathbf{\Omega}_j(t)$  at time  $t$  are obtained by linear interpolation between two

snapshots. Using equation (24),  $\psi^e(r < R_{200}, \mathbf{\Omega})$  can be reconstructed from the coefficients  $b_m^e$ .

Section 4.2 makes extensive use of equations (35)-(36) for each halo in our simulations to characterise statistically these two fields.

### 3.3 From flux densities to the 5D source

The description of the source term  $s^e$  involves time dependent coefficients  $c_{\alpha\ell m'}^{\ell m}(t)$ . Their computation from the particles coordinates is quite straightforward and as shown in the previous sections, the different margins can be recovered through the manipulation of these coefficients. Yet a projection of the source on an *a priori* basis is a complex operation. Here this projection aims at describing a 5D space for which little is known. As shown in the following sections, the distribution of incidence angles is quite smooth, while the distribution of velocities appear to be easily parametrized by gaussians. The 5D basis presented in the current paper induces little bias, but it is very likely that a more compact basis exists and that the size of the expansions chosen can be reduced in the future.

Because of this large amount of information contained in the source, it is not always convenient to relate coefficients or their correlations to physical quantities, like the mass flux or the flux density of energy. An alternate description of the source term was presented in Aubert et al. (2004) with the following ansatz:

$$s^e(\mathbf{r}, \mathbf{v}, t) = \sum_{\mathbf{m}} Y_{\mathbf{m}}(\mathbf{\Omega}) \frac{\hat{\omega}_{\rho, \mathbf{m}} (2\pi)^{-3/2}}{\det|\hat{\omega}_{\rho\sigma\sigma, \mathbf{m}}/\hat{\omega}_{\rho, \mathbf{m}}|} \times \quad (37)$$

$$\exp \left[ -\frac{1}{2} \left( \mathbf{v} - \frac{\hat{\omega}_{\rho\mathbf{v}, \mathbf{m}}}{\hat{\omega}_{\rho, \mathbf{m}}} \right)^{\top} \left( \frac{\hat{\omega}_{\rho\sigma\sigma, \mathbf{m}}}{\hat{\omega}_{\rho, \mathbf{m}}} \right)^{-1} \left( \mathbf{v} - \frac{\hat{\omega}_{\rho\mathbf{v}, \mathbf{m}}}{\hat{\omega}_{\rho, \mathbf{m}}} \right) \right].$$

This representation of the source is by construction consistent with the first two velocity moments:

$$\int d^3\mathbf{v} s^e(\mathbf{r}, \mathbf{v}) = \varpi_{\rho}(\mathbf{r}), \quad \int d^3\mathbf{v} \mathbf{v} s^e(\mathbf{r}, \mathbf{v}) = \varpi_{\rho\mathbf{v}}(\mathbf{r}), \quad (38)$$

while

$$\int d^3\mathbf{v} (v_i - \frac{\varpi_{\rho\mathbf{v}, i}}{\varpi_{\rho}}) (v_j - \frac{\varpi_{\rho\mathbf{v}, j}}{\varpi_{\rho}}) s^e(\mathbf{r}, \mathbf{v}) =$$

$$\varpi_{\rho\sigma_i\sigma_j}(\mathbf{r}) - \frac{\varpi_{\rho\mathbf{v}}(\mathbf{r})^2}{\varpi_{\rho}(\mathbf{r})} + \sum_{\mathbf{m}} Y_{\mathbf{m}}(\mathbf{\Omega}) \delta(r - R_{200}) \frac{\hat{\omega}_{\rho\mathbf{v}, \mathbf{m}}(t)^2}{\hat{\omega}_{\rho, \mathbf{m}}(t)},$$

$$\approx \varpi_{\rho\sigma_i\sigma_j}(\mathbf{r}). \quad (39)$$

Obviously, the third moment is not fully recovered from the Ansatz given by equation (38). This example should be taken as an illustration and highlights the possibility of building a source term from its moments. It is not unique and more realistic expressions could be found, which satisfies higher moments of the source. Still, the successive measurements on the sphere of the flux density of mass  $\varpi_{\rho}$ , momentum  $\varpi_{\rho\mathbf{v}}$  and velocity dispersion  $\varpi_{\rho\sigma\sigma}$  allow a coherent description of the infall of matter. Unlike the coefficients, these flux densities are easier to interpret since they describe physical quantities and are directly involved in specific dynamical processes (see table 3.3). Furthermore, these three flux densities are easily expressed in terms of coefficients  $c_{\alpha\ell m'}^{\ell m}(t)$ , or more precisely in terms of a *subset* of the source's coefficients, implying a smaller number of computations relative to a complete calculation of  $c_{\alpha\ell m'}^{\ell m}(t)$ . Finally these flux densities are particularly suited to the regeneration of synthetic environments. As shown in appendix F, synthetic spherical maps can be generated from the

two-point correlations and cross-correlations of these fields. Such environments would be consistent with the measurements in simulations and will allow us to easily embed simulated galaxies or halos in realistic environments as a function of time.

The expression given in equation (38) has one important drawback : it is not of the form of equation (12), i.e. it would require a reprojecton over a linear expansion for a dynamical propagation. Nevertheless, its compacity makes it easier to compute than the full set of coefficients and the associated strategy would be 1) to measure the flux densities from the simulation, 2) build a source term from e.g. the Equation (38) and 3) project over an appropriate 5D basis when needed, i.e. when the source is used as an input to the analytic description of the haloes dynamics.

The following sections will make intensive use of the coefficients described by the equation (35) and equation (36). In particular, it will shown how the manipulation of these coefficients allow to recover relevant physical quantities. In the current paper, only the first moment of the source, the flux density of mass  $\varpi_{\rho}$  together with the external potential, will be fully assessed. The kinematical properties of the accreted material will in particular be investigated. The complete characterization of the  $\mathbf{c}(\mathbf{t})$  coefficients is beyond the scope of the current paper and will be completed in paper III. The full measurements of these 11 fields required by Eq. 38 and the comparisons between the two expressions of the source will also be assessed in this future paper as well. Appendix G1 and G2 describe how the other moments, the flux density of momentum  $\varpi_{\rho\mathbf{v}}$  or the flux density of energy  $\varpi_{\rho\sigma^2}$  may be recovered from the source expansion.

### 3.4 A template halo

Let us first as an illustration apply the whole machinery to one typical halo. At  $z=0$ , this 'template' halo has a mass  $M$  of  $3.4 \cdot 10^{13} M_{\odot}$ , with a virial radius  $R_{200}$  of  $800 h^{-1}$  kpc. The corresponding circular velocity is  $V_{c0} = 600$  km/s. Its accretion history is shown in figure 5 for  $z < 1$ . Each point on the azimuth-time diagram represents one particle of the simulation passing through the virial sphere at a given azimuth and at a given time. Temporal space has been sampled using 15 equally spaced bins between  $z=1$  and  $z=0$  (see bottom panel in figure 5). For each time step, the expansion coefficients  $c_{\alpha, \mathbf{m}}^{\mathbf{m}'}(t)$  are computed from equation (35). The Gaussian basis  $g_{\alpha}(v)$  involved 25 functions with means  $\mu_{\alpha}$  equally distributed from  $v=0$  to  $v=1.5$  in  $V_{c0}$  units and with an r.m.s  $\sigma = 0.03$ . The harmonic expansions were carried up to  $\ell = 50$  in position space and up to  $\ell' = 25$  in velocity space.

#### 3.4.1 Advected mass: angular space ( $\varpi_{\rho}$ )

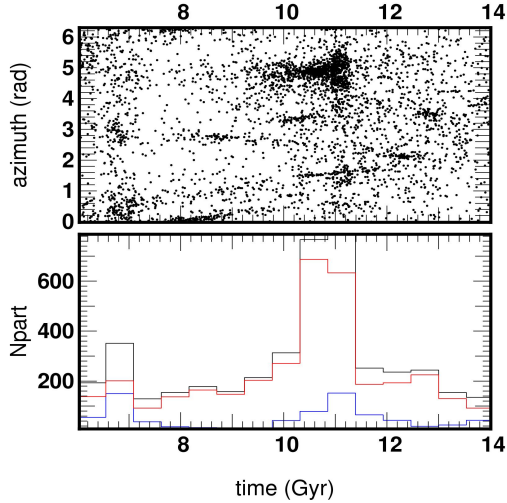
The template halo accretes an object at  $t_s=11$  Gyr (where  $t=14$  Gyr stands for  $z=0$ ) adding  $7.5 \cdot 10^{12} M_{\odot}$  to the system during a  $\sim 1$  Gyr interval. The corresponding spherical flux density field,  $\varpi_{\rho}(\mathbf{\Omega}, t_s)$  is shown in Fig. (6). It represents the distribution of accreted particles as seen from a halo-centric point of view. The field  $\varpi_{\rho}(\mathbf{\Omega}, t_s)$  has been reconstructed from the coefficients (see equation (35)). It reads:

$$\varpi_{\rho}(\mathbf{\Omega}, t) = \int d\mathbf{\Gamma} dv v^2 s^e(v, \mathbf{\Omega}, \mathbf{\Gamma}, t) = \sum_{\mathbf{m}} a_{\mathbf{m}}(t) Y_{\mathbf{m}}(\mathbf{\Omega}). \quad (40)$$

Since

	flux density, $\varpi$	Flux, $\Phi$	Motivation
Mass	$\rho v_r$	$dm/dt$	heating & cooling,
Angular momentum	$\rho v_r \mathbf{r} \times \mathbf{v}$	$dL/dt$	warp, shape of halos,
Kinetic energy	$\rho v_r \sigma_i \sigma_j$	$dE/dt$	virialized objects,
Shear	$\rho v_r (\partial v_j / \partial x_i + \partial v_i / \partial x_j)$	$dc/dt$	tidal field,
Vorticity	$\rho v_r \nabla \times \mathbf{v}$	$d\omega/dt$	anisotropic accretion.

**Table 1.** Description of the various flux densities. The first 10, together with the external potential are sufficient to characterize fully the environment as shown in Section 3.3.



**Figure 5.** An example of accretion history. *Top* : Azimuth-time diagram. Each point in the diagram represents one particle passing through the virial sphere at a given azimuth (y-axis) at a given time measured from the Big-Bang (x-axis). *Bottom* : the distribution of particles' crossing instants (in black). Time increases from left to right. The infalling (resp. outflowing) particles distribution is shown in red (resp. blue).

$$\int d\Gamma Y_{\ell',m'} = \sqrt{4\pi} \delta_{\ell'0} \delta_{m'0}, \quad \text{and} \quad \int dv v^2 g_\alpha(v) = \mu_\alpha^2 + \sigma^2. \quad (41)$$

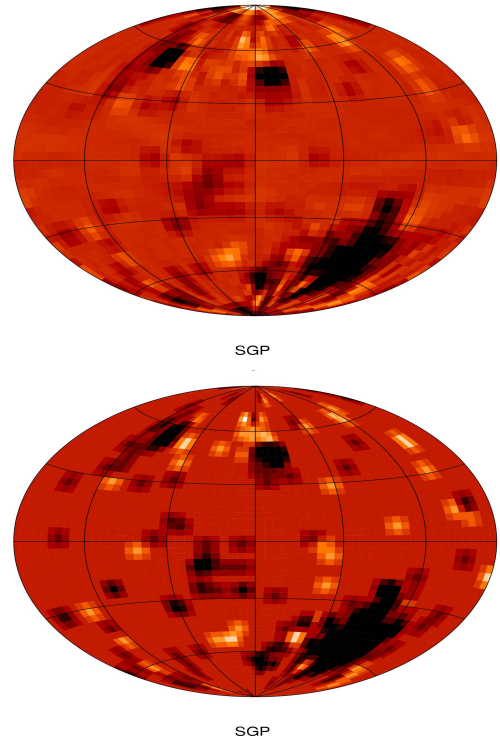
It follows that

$$a_m(t) = \sqrt{4\pi} \sum_\alpha (\mu_\alpha^2 + \sigma^2) c_{\alpha,0}^m(t), \quad (42)$$

allowing us to recover  $\varpi_\rho(\Omega, t_s)$ . Also shown, the same field but computed this time using directly the angular distribution of particles as described in Aubert et al. (2004) (see below). All the major features are well reproduced by the expansion coefficients, equations (35)-(42). Clearly, an object is ‘falling’ through the virial sphere. It is straightforward to obtain the angular power spectrum  $C_\ell^{\varpi\rho}$  from the  $c_{\alpha,m}^m$  coefficients via the definition of  $a_{\ell m}$  in equation (42) :

$$C_\ell^{\varpi\rho} = \frac{1}{4\pi} \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2. \quad (43)$$

The angular power spectrum of  $\varpi_\rho(\Omega, t_s)$ , derived from the expansion, equation (35), is shown in Fig. (7). From the positions and velocities of particles it is also possible to evaluate  $\varpi_\rho(\Omega, t_s)$  on an angular grid and recover the angular power spectrum ‘directly’. The agreement between the two  $C_\ell^{\varpi\rho}$  is good, though for the smallest scales ( $\ell \geq 30$ ), the power spectrum computed from the coefficients is slightly larger than the one derived directly from the particles. This may be explained by the fact that a grid sampling tend to smooth the actual  $\varpi_\rho$  field. As a consequence the amplitude of



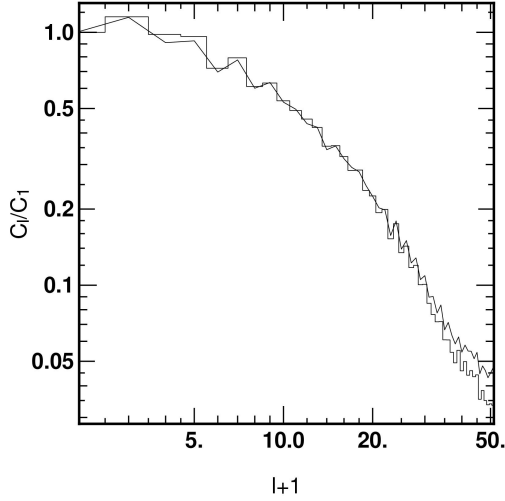
**Figure 6.** An example of a flux density reconstructed from the coefficients  $c_{\alpha,m}^m$ : the mass flux density,  $\rho v_r(\Omega)$ . It represents the angular distribution of incoming mass as seen from a halo-centric point of view. Here  $t_s \sim 11$  Gyr. Light regions correspond to strong infall while darker regions stand for low accretion and outflows. *Top* : the spherical field obtained directly from the spatial distribution of particles. *Bottom* : the reconstructed spherical field from the coefficients, equation (35).

small scales fluctuations is decreased, leading to a smaller  $C_\ell^{\varpi\rho}$ . A more complete discussion on harmonic convergence can be found in appendix A. For a given  $\ell$ , the corresponding angular scale is  $\pi/\ell$  in radians.

Note that the coefficients  $a_{00}$  are closely related to the accretion field averaged over all directions,  $\Phi^M(t)$ , defined by :

$$\Phi^M(t) \equiv \overline{\varpi_\rho} = \frac{1}{4\pi} \int d\Omega \rho v_r(\Omega, t) = \frac{a_{00}}{\sqrt{4\pi}}. \quad (44)$$

Measuring  $a_{00}(t)$  amounts to measuring the accretion flux density, *i.e.* the quantity of dark matter accreted per unit surface and per unit time.



**Figure 7.** The angular power spectrum,  $C_\ell/C_1$ , (equation (43)) of the distribution of incoming matter,  $\varpi_\rho(\Omega)$  (shown in figure 6) at  $t \sim 11$  Gyr, for our template halo. For a given  $\ell$ , the corresponding angular scale is  $\pi/\ell$ . The histogram corresponds to the power spectrum derived directly from the particles' angular distribution. The solid line is the power spectrum reconstructed from the coefficients.

### 3.4.2 Advected mass: velocity space

Integration over the sphere leads to the distribution of accreted matter in velocity space :

$$\rho v_r(\mathbf{\Gamma}, v, t) = \int d\Omega s^e(v, \Omega, \mathbf{\Gamma}, t), \quad (45)$$

$$= \sqrt{4\pi} \sum_{\alpha, m'} c_{\alpha, m'}^0 g_\alpha(v) Y_{m'}(\mathbf{\Gamma}). \quad (46)$$

Projections over  $\Gamma_2$  and  $v$  gives the probability distribution of the incidence angle  $\Gamma_1$ ,  $\vartheta(\Gamma_1, t)$ , defined as:

$$\begin{aligned} \vartheta(\Gamma_1, t) &= \int d\Gamma_2 dv v^2 \rho v_r(\mathbf{\Gamma}, v, t), \\ &= 2\pi\sqrt{4\pi} \sum_{\alpha, \ell'} c_{\alpha, \{\ell', 0\}}^0 (\mu_\alpha^2 + \sigma^2) Y_{\ell', 0}(\mathbf{\Gamma}). \end{aligned} \quad (47)$$

The impact parameter  $b$  of an incoming particle (measured in units of the virial radius) is related to  $\Gamma_1$  by

$$\frac{b}{R_{200}} = \sin(\Gamma_1), \quad (48)$$

therefore the probability distribution of impact parameters,  $\vartheta(b)$ , is easily deduced from equation (47). At  $t \sim 11$  Gyrs, the  $\vartheta(b)$  computed from the source coefficients is compared to that derived directly from the velocities of particles in Fig. (8). Note that for pure geometrical reasons small impact parameter  $b$  are less likely since there is only one trajectory passing through the center while there is a whole cone of trajectories with  $b \neq 0$ . As a consequence errors are intrinsically larger for small values of  $b$ . The reconstruction from the source coefficients is clearly adequate. In this example, the high probability for infalling particles to have a small impact parameter ( $b < 0.5$ ) imply that velocities are strongly radial. The object 'dives' into the halo's potential well.

Projection over  $\Gamma_1$  leads to the probability distribution of particles velocities,  $\varphi(v, t_s)$ , as they pass through the virial sphere. The PDF  $\varphi(v, t)$  is defined as:

$$\varphi(v, t) \equiv v^2 \int d\Omega d\Gamma s^e(v, \Omega, \mathbf{\Gamma}, t). \quad (49)$$

Here the  $v^2$  weighting accounts for the fact that we are interested in the probability distribution of measuring a velocity,  $v$  within  $dv$ . Using coefficients, it follows that:

$$\varphi(v, t) = 4\pi \sum_{\alpha} v^2 g_\alpha(v) c_{\alpha, 0}^0. \quad (50)$$

The reconstructed velocity distribution is also shown in Fig. (8). It reproduces well the actual velocity distribution. For this specific halo, the satellite is being accreted with a velocity of  $0.75 V_{c0}$ .

The correlation between the incidence angle  $\Gamma_1$  and the velocity's amplitude  $v$  may be studied by integrating  $\rho v_r(\mathbf{\Gamma}, v, t)$  over  $\Gamma_2$  only. The distribution function,  $\varphi(\Gamma_1, v)$ , of particles in the  $(\Gamma_1, v)$  subspace is defined by:

$$\begin{aligned} \varphi(\Gamma_1, v) &\equiv \int d\Gamma_2 dv v^2 \rho v_r(\mathbf{\Gamma}, v, t), \\ &= 2\pi\sqrt{4\pi} \sum_{\alpha, \ell'} c_{\alpha, \{\ell', 0\}}^0 g_\alpha(v) Y_{\ell', 0}(\Gamma_1, 0). \end{aligned} \quad (51)$$

Given the relation equation (48), the correlation  $\varphi(b, v)$  between the impact parameter and the velocity's amplitude is easily obtained. The  $\varphi(b, v)$  distribution is shown in Fig. (8). Again, note that  $\varphi(b, v)$  represents *an excess* probability of finding an impact parameter  $b$  (with a velocity  $v$ ) compared to isotropy. In this specific example, no real correlation may be found between the two quantities. Finally, the integration of  $\varphi(b, v)$ ,  $\varphi(v, t)$  and  $\vartheta(\Gamma_1)$  over their respective space leads to the same quantity, namely the integrated flux  $\Phi^M(t)$ .

### 3.4.3 External potential

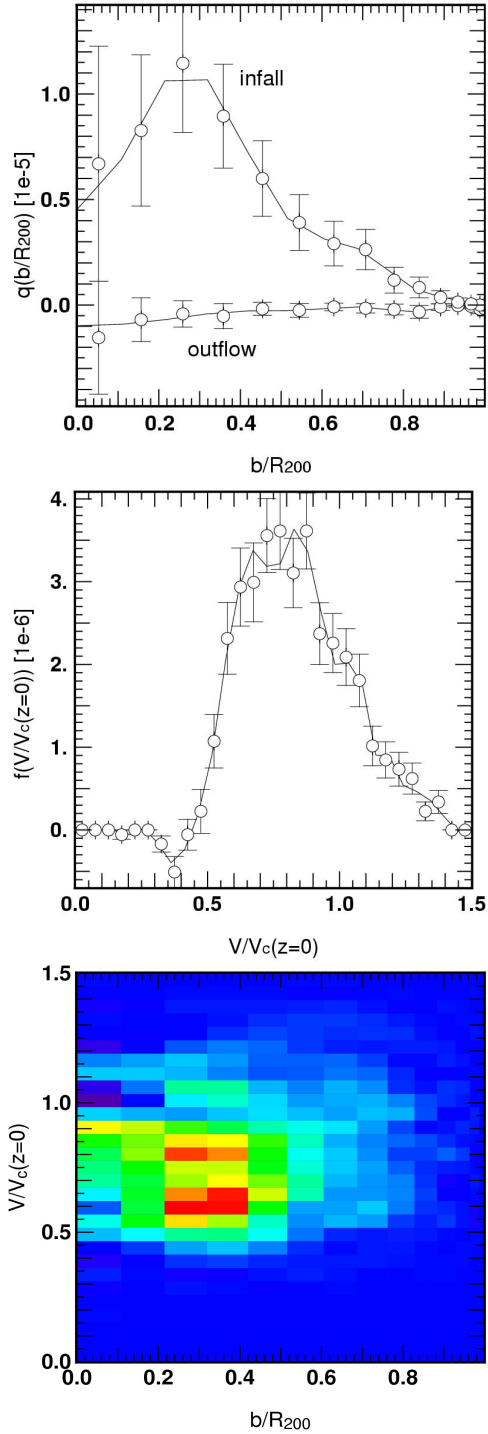
The final field needed on the virial sphere is the external tidal field created by the dark matter distribution around the halo.

Using equation (36), the external potential  $\psi^e(\Omega, t)$  is easily computed from the positions of external particles, having restricted the sampling to particles within a 4 Mpc (physical) sphere centered on the halo. The position of external particles are linearly interpolated at a given measurement time. The  $b_m$  coefficients for the template halo are computed at  $t_s \sim 7$  Gyrs (measured from the Big Bang). The reconstructed  $\psi^e(\Omega, t)$  field is shown in Fig. (9) along with the modulus of the advected mass  $|\rho v_r(\Omega)|$ . The two reconstructions were restricted to harmonics  $\ell \leq 20$ .

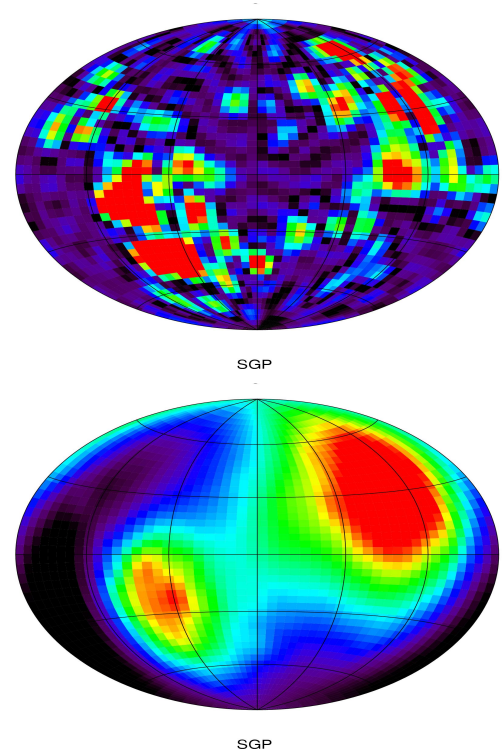
The two spherical fields show the same main features. However, almost no small scales features is seen in the map of the external potential even though they have the same resolution. Since the gravitational potential is known to be smoother than the associated density and is dominated by the global tidal field, it is not surprising that  $\psi^e(\Omega, t)$  appears smoother than the advected mass field  $|\varpi_\rho(\Omega)|$ .

The potential's angular power spectrum may also be computed by replacing  $a_{\ell m}$  by  $b_{\ell m}$  in equation (43) (see Fig. (10)). The power spectrum of the potential,  $C_\ell^{\varpi_\rho}$ , sharply decreases with  $\ell$ , while  $C_\ell^{\rho v_r}$  has a gentler slope. Large scales are clearly more important for the potential than for the advected mass. Furthermore,  $C_\ell^{\psi^e}$  systematically peaks for even  $\ell$  values, reflecting the 'even' symmetry of the potential measured on the sphere.

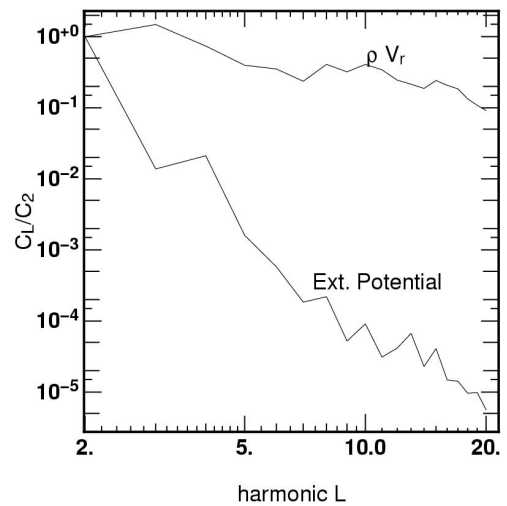




**Figure 8.** *Top:* Excess probability distribution of impact parameter  $b$ ,  $\vartheta(b)$ , derived from the  $c_{\alpha, m'}^m(t)$  source coefficients (equation (35)) of our template halo at  $t_s \sim 11$  Gyr (line). The histogram corresponds to the same distribution derived directly from the particles' positions and velocities. Error bars stand for  $3\sigma$  errors. The impact parameters are given in units of  $R_{200}$ . The Y-axis unit is  $5 \cdot 10^9 M_\odot / \text{kpc}^2 / \text{Myr}$ . Infall is mainly radial. *Middle:* The velocity distribution of particles,  $\varphi(v)$ , accreted at  $t_s \sim 11$  Gyr, for our template halo. Velocities are expressed in terms of the circular velocity at  $z=0$ . The Y-axis unit is  $5 \cdot 10^9 M_\odot / \text{kpc}^2 / \text{Myr}$ . The histogram corresponds to the velocity distribution obtained directly from the particle velocities. The solid line is the reconstructed distribution from the source coefficients. *Bottom:* the probability distribution,  $\varphi(b, v)$ , of particles in the  $b-v$  subspace. Units are the same as above. Red/blue stand for high/low densities. No correlation is found between  $b$  and  $v$  for this specific example.



**Figure 9.** A comparison between the external potential,  $\psi^e(\Omega)$ , and the modulus of the flux density of matter,  $|\rho v_r(\Omega)|$ . The measurement is made at  $t \sim 7$  Gyrs (measured from the Big Bang) on our template halo. The two fields were respectively reconstructed from  $c_{\alpha, m'}^m$  and  $b_m$  coefficients with  $\ell_{\max} \leq 20$ . Even though the two fields are similar and exhibits a strong quadrupolar component,  $\psi^e(\Omega)$  is smoother than  $\rho v_r(\Omega)$ . It is expected that the corresponding expansion coefficients be statistically correlated.



**Figure 10.** A comparison between the angular power spectrum of  $\rho v_r(\Omega)$  and  $\psi^e(\Omega)$  for our template halo (the two fields are shown in Fig. (9)). The two power spectra  $C_\ell$  are normalized by  $C_2$ , i.e the quadrupole contribution. The slope of the potential's power spectrum is clearly stronger. Large scales (i.e. small  $\ell$  values) dominate the angular distribution of  $\psi^e(\Omega)$ , as expected.

#### 4 SIMULATION SAMPLE & STATISTICAL BIASES

Section 3.4 details the measurement strategy for a given typical halo. We are now in a position to reproduce the above measurements for all the halos of the simulation sample. Let us first describe in turn the construction of our sample, and the corresponding biases, which constrain our ability to convert a large set of simulations into the statistics of the source.

##### 4.1 Simulations

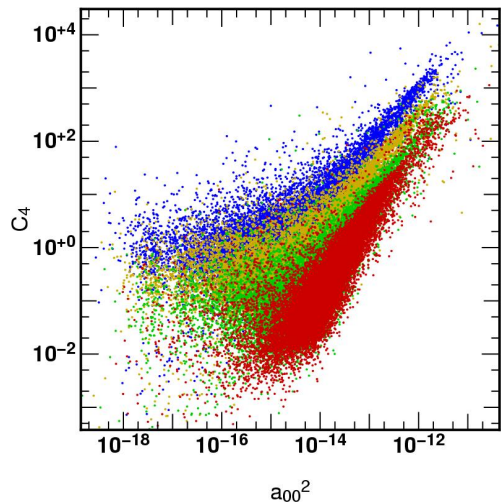
In order to achieve a sufficient sample and ensure a convergence of the measurements, a set of  $\sim 500$  simulations was produced as discussed in Aubert et al. (2004). Each of them consists of a  $50 h^{-1} \text{Mpc}^3$  box containing  $128^3$  particles. The mass resolution is  $5 \cdot 10^9 M_\odot$ . A  $\Lambda$ CDM cosmogony ( $\Omega_m = 0.3$ ,  $\Omega_\Lambda = 0.7$ ,  $h = 0.7$  and  $\sigma_8 = 0.928$ ) is implemented with different initial conditions. These initial conditions were produced with GRAFIC (Bertschinger (2001)) where a BBKS (Bardeen et al. (1986)) transfer function was chosen to compute the initial power spectrum. The initial conditions were used as inputs to the parallel version of the tree code GADGET (Springel et al. (2001)). The softening length was set to  $19 h^{-1} \text{kpc}$ .<sup>3</sup> The halo detection was performed using the halo finder HOP (Eisenstein & Hut (1998)). The density thresholds suggested by the authors ( $\delta_{\text{outer}} = 80$ ,  $\delta_{\text{saddle}} = 2.5\delta_{\text{outer}}$ ,  $\delta_{\text{peak}} = 3\delta_{\text{outer}}$ ) were used.

##### 4.2 Selection criteria

As shown in Aubert et al. (2004) the completion range in mass of the simulations spans from  $3 \cdot 10^{12} M_\odot$  to  $3 \cdot 10^{14} M_\odot$ . Since the emphasis is on  $L_*$  galaxies, the survey is focused mainly on galactic halos and light clusters, only haloes with a mass smaller than  $10^{14} M_\odot$  at  $z=0$  were considered. The interest is for halos already ‘formed’, *i.e.* which will not experience major fusions anymore. To satisfy these requirements the focus is on the last 8 Gyrs (redshifts  $z < 1$  in a  $\Lambda$ CDM cosmogony). Since the history of a given halo is followed by finding its most massive progenitor, it is required not to accrete more than half its mass in a two-body fusion. As a final safeguard a halo is rejected if it accretes more than  $5 \cdot 10^{12} M_\odot$  between two timesteps (*i.e.* per 500 Gyrs, see next subsection). This mass corresponds approximatively to the smallest haloes considered at  $z = 0$ . The final range of mass of haloes which satisfy these criteria is  $\sim 5 \cdot 10^{12} M_\odot - 10^{14} M_\odot$ , the fraction of rejected haloes being  $\sim 20\%$ . Clearly, such *a priori* selection criteria will modify the distributions of measured values and the related biases may be difficult to predict. For instance, the Fig. (11) shows the scatter plot of the contribution of  $\pi/(\ell = 4) = 45$  degrees fluctuations to  $\varpi_\rho$  field vs  $a_{00}$ , *i.e.* the accretion rate. It appears from this plot that modifying the threshold for the accretion will modify the average angular scale of  $\varpi_\rho$  in a non trivial way. Since only a small fraction of haloes is rejected, the biases are expected to be moderate, but as for now their impact cannot be estimated accurately on the average source or its moments.

Let us emphasize that the above selection criteria should be added to those corresponding to the simulations themselves. Aside from the fact that a  $50 \text{Mpc}^3 h^{-1}$  box size implies a limited range

<sup>3</sup> A second set of simulations with  $2^3$  (resp.  $2^6$ ) the resolution in mass was carried in order to investigate the convergence of some measurements (see Sec. ??).



**Figure 11.** Scatter plot of  $C_4^{\varpi_\rho}$  versus  $a_{00}^2$  measured at lookback times  $t = 7.8$  (red),  $5.6$  (green),  $4$  (yellow) and  $2.9$  (blue) Gyrs. The quantity  $a_{00}$  scales as the average accretion rate of the haloes while  $C_4^{\varpi_\rho}$  scales as the contribution of  $\ell = 4$  structures in the flux density of mass measured on the sphere. This plot illustrates how a threshold on the accretion rate affects in a non trivial way the typical clustering measured for  $\varpi_\rho$ . In particular, one should note how  $C_4^{\varpi_\rho}$  remains constant at recent times for low accretion rates.

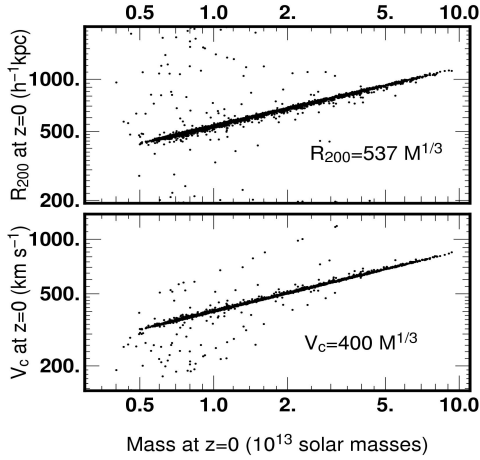
of mass, the universe described in these simulations is more homogeneous than it should be, since each box must satisfy a given mean density. In other words, the probability of rare events is reduced. This effect should not influence the number of haloes with high accretion rate, since strong accretions are rejected a priori. On the other hand, it should influence the number of objects which experience low accretion history, which are probably less numerous in our simulations than in larger simulated volumes since voids are less likely. Furthermore, the intrinsic mass resolution sets a minimum accretion rate equals to one particle mass ( $5 \cdot 10^9 M_\odot$ ) per time interval. One could imagine an object with a mass smaller than the particle’s mass which would not be included in the simulation at the current resolution. Furthermore, an object with a mass equals to a few times the minimum mass is be considered as diffuse accretion. Finally this mass resolution is related to the spatial resolution, which limits intrinsically the angular description of fluxes on the virial sphere. For a given type of simulation, all these effects cannot be avoided and reduce the representability of the following measurements.

In short, this strategy involves a bias in mass, in redshift, in resolution and in strength of merging event. However these biases should only influence somewhat extreme realisations (related to *e.g.* very low accretion or equal mass mergers) of the source or the external potential and since the focus is on the typical scales, presumably related to moderate interactions, they should hopefully not significantly affect the measurements.

##### 4.3 Reduction procedure

In the following discussion most of the distances (resp. velocities) will be expressed as functions of the virial radius  $R_{200}$  (resp. the circular velocity  $V_c$ ) measured at  $z = 0$ . These quantities are related to the halo’s final mass by





**Figure 12.** Virial radii ( $R_{200}$ ) and circular velocities ( $V_c$ ) as functions of halos final masses. The quantities have been measured at redshift  $z = 0$ . Scaling relations between  $R_{200}$  or  $V_c$  and the final mass are also given.

$$V_c = \sqrt{\frac{GM}{R_{200}}}. \quad (52)$$

The mass-dependence of  $R_{200}$  and  $V_c$  are given in Fig. (12) and may be fitted by:

$$R_{200} = 537M^{1/3}, \quad V_c = 400M^{1/3}, \quad (53)$$

where  $R_{200}$  is expressed in ( $h^{-1}$ kpc),  $V_c$  in km/s and  $M$  in units of  $10^{13}M_\odot$ . In Equation (53),  $R_{200}$  and  $V_c$  appear to be strongly correlated to the final masses of haloes. Since the selection criteria are quite restrictive, most of the haloes experience the same relatively quiet history of accretion and account for the lack of scatter.

The simulations in that redshift interval involved 15 snapshots sampled with  $\Delta(\log z) = \text{cst}$  for  $z \leq 1$  down to  $z = 0.1$  plus a snapshot at  $z = 0$ . The gap between the last snapshot and the second to the last is nearly 1.4 Gyr. As a consequence, the assumption of ballistic trajectories is not valid anymore (see appendix). Simulations were re-sampled in 15 bins distributed regularly in *time* (i.e not in redshift) using the procedure described in Section 3.2: the corresponding time step is  $\sim 500$  Gyrs. To take in account the last gap, results obtained from the last three ‘new’ bins (which cover the last 1.4 Gyr) were averaged into a single bin centered on 0.8 Gyr.

The source coefficients,  $c_{\alpha, m'}^m(t)$ , were computed following the procedure described in section 3.4. Maximum harmonic orders were set to  $\ell_{\text{max}} = 50$  for the position-angular description. For a typical halo with  $R_{200} = 500$  kpc,  $\ell_{\text{max}} = 50$  corresponds to a spatial scale of 30 kpc, i.e. equals to 1.5 times the spatial resolution of the simulation. The harmonic description of the velocities angular dependence is restricted to  $\ell'_{\text{max}} = 15$ . The velocity amplitude is projected on a gaussian basis which involves 25 functions regularly spaced from  $v/v_c = 0$  to  $v/v_c = 1.5$  with an r.m.s of 0.03. These parameters allow a satisfying reproduction of distributions computed from particles.

The external potential coefficients,  $b_m(t)$ , were computed following the procedure described in section 3.4. Only particles within a 4 Mpc physical sphere centered on the halo are taken in account. Maximum harmonic orders were set to  $\ell_{\text{max}} = 20$ .

A set of 100 simulations have been fully reduced allowing us to compute  $c_{\alpha, m'}^m(t)$  and  $b_m(t)$  for 15000 halos. Since a well de-

finied, (if only biased) sample of histories of haloes was constructed in our simulations, it may be projected on our basis, to compute the external potential and the flux density of mass, following Section 3.4. Let us now characterize the corresponding coefficients, via one point (Section 5) and two points (Section 6) statistics.

## 5 ONE POINT STATISTICS

In this section, let us describe the evolution and the statistical distributions of the global properties (i.e. integrated over the sphere) of the source and the potential. Let us discuss the evolution of the mean potential, of the mass flux  $\Phi^M(t)$  and the kinematical properties of  $s^e$  via the velocity distribution  $\phi(v)$  and the impact parameter distribution  $\vartheta(b)$ .

Let us describe in turn the statistical PDF, mean and variance of the integrated fluxes, their corresponding flux densities, and finally their mean kinematical features, following the steps of section 3.4.

### 5.1 Mean External potential

The mean external potential on the sphere is actually somewhat meaningless but is being used as a normalization value for potential fluctuations (see section 6.1). Because of isotropy, the mean potential is seen as a monopole and only the  $b_{00}(t)$  coefficient is statistically different from zero. Furthermore, following equation (24), the three dimensional potential component induced by the monopole is a constant potential throughout the sphere volume. As a consequence, it influences the halo’s dynamics only through its temporal variation.

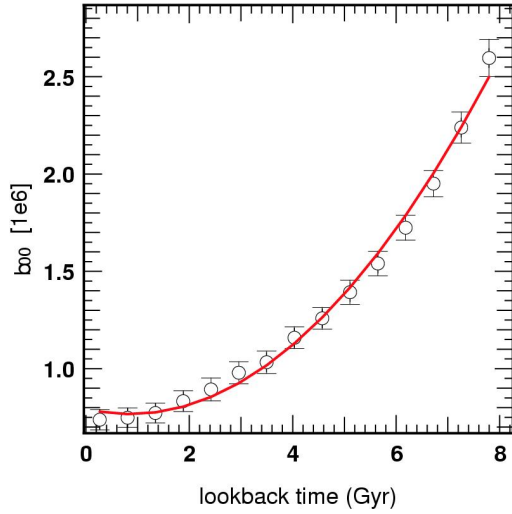
The time evolution of the  $\langle b_{00} \rangle$  coefficient is given in Fig. (13). The  $b_{00}(t)$  distribution exhibits a tail due to large  $-b_{00}$  values and is better fitted with a log-normal distribution than with a Normal distribution (see Fig. C1). Hence  $\langle b_{00} \rangle$  stands for the most probable value of the log-normal fitting distribution.<sup>4</sup> The evolution shown in Fig. (13) reflects the measurement procedure. Given that the potential is computed from all the particles contained within a fixed physical volume, the overall expansion implies that particles tend to exit the measurement volume with time. In other words, the average density in the measurement volume decreases with time. This effect leads naturally to the decline of the average potential within the virial sphere due to external material.

### 5.2 Mass flux : $\Phi^M(t)$

At each time-step, the  $a_{0,0}$  distribution is fitted by a Gaussian function with mean  $\langle a_{0,0} \rangle$  (see also Fig. (C3)). This gaussian hypothesis is clearly verified for low redshifts while strong accretions events give rise to a tail in the  $a_{0,0}$  distribution at high  $z$ . At these epochs (lookback time  $t > 7$  Gyr) the gaussian fit tends to slightly overestimate the mode position. Yet the gaussian hypothesis remains a good approximation of the distribution while the time evolution of the gaussian mean value  $\langle a_{0,0} \rangle(t)$  represents well the evolution of the mode of  $a_{0,0}$ .

The time evolution of the average flux of matter through the sphere,  $\Phi^M(t) = \overline{\varpi_\rho}$  is directly derived from the evolution of the

<sup>4</sup> Since the measured distribution is quite peaked, fits made with a Normal distribution (not shown here) return a very similar time evolution of the mode position.



**Figure 13.** The time evolution of the monopole component of the external potential,  $b_{00}(t)$ . The time evolution is fitted by a second order polynomial  $-b_{00}(t) = 35948 * t^2 - 61480.7 * t + 793067$ . Look back time  $t$  is expressed in Gyrs while  $b_{00}$  coefficients are expressed in units of  $GM/R$ .  $M$  is expressed in  $10^{10} M_{\odot}$ ,  $R$  in  $\text{kpc} h^{-1}$  and  $G=43007$  in internal units.

monopole (see equation (44)) and is shown in Fig. (14). It can be fitted by:

$$\Phi^M(t) \equiv \langle \overline{\omega_{\rho}} \rangle(t) = -0.81t^3 + 10.7t^2 - 19.3t + 17.57, \quad (54)$$

where  $\Phi^M(t)$  is in units of  $M_{\odot}/\text{Myr}/\text{kpc}^2$  and look-back time  $t$  is expressed in Gyr. As expected, the average quantity of material accreted by halos decreases with time. For  $z < 1$ , a large fraction of the objects of interest in are already ‘formed’ and only gain matter through the accretion of small objects or diffuse material. In a hierarchical scenario, such source of matter become scarcer, inducing a decrease in the accretion rate. Furthermore, recall that  $\Phi^M(t)$  is measured as a net flux, i.e. the outflowing material may cancel a fraction of the infalling flux. Therefore, the decrease with time may also be the consequence of an increasing contribution of outflows: the measurement radius  $R_{200}(z=0)$  becomes the actual virial radius of the halo as time goes by, i.e. the radius where the inner material is re-processed and where outflows are susceptible to be detected.

As a check the average mass accretion history (MAH hereafter) of the haloes was computed. The MAH  $\Psi(t)$  is defined as:

$$\Psi(t) \equiv \frac{M(t)}{M(z=0)}, \quad (55)$$

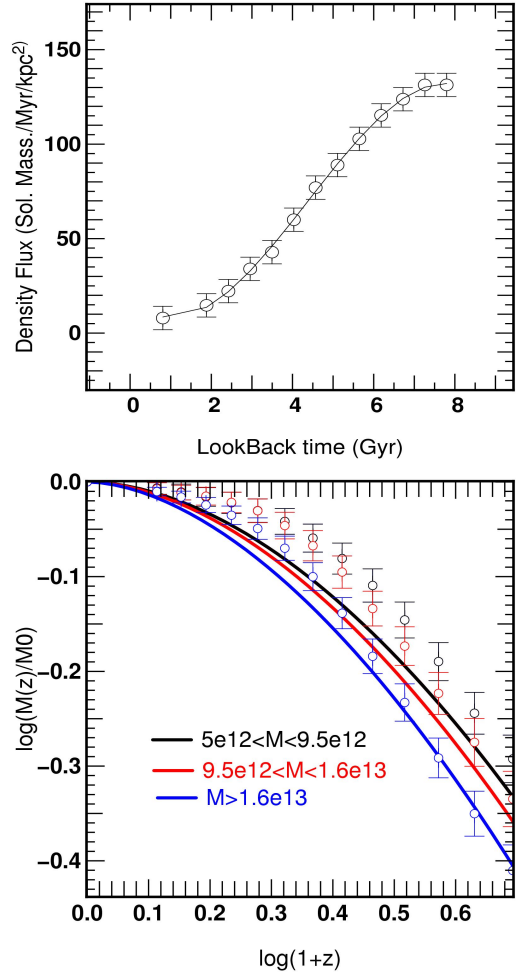
where  $M(z)$  is the halo’s mass at a given instant. Using the extended Press-Schechter formalism, van den Bosch (2002a) showed that halos have an universal MAH, fitted by the following formula:

$$\log(\Psi(M(z=0), t)) = -0.301 \left[ \frac{\log(1+z)}{\log(1+z_f)} \right]^{\nu}, \quad (56)$$

where  $z_f$  and  $\nu$  are two parameters which depend on the considered class of mass only. For each halo, its mass evolution  $M(t)$  was computed from its final mass  $M(z=0)$  and its integrated flux of matter  $\Phi^M(t)$ :

$$M(t) = M(z=0) - 4\pi R_{200}^2 \int_{t=0}^t dt \Phi^M(t). \quad (57)$$

From equation (55)  $\Psi(t)$  was computed for each halo. The *median*



**Figure 14.** *Top:* Time evolution of the average flux density of matter through the virial sphere,  $\langle \Phi^M(t) \rangle = \langle \overline{\omega_{\rho}} \rangle(t)$  (symbols). Bars stand for  $3\sigma$  errors. Here  $\Phi^M(t)$  is computed directly from  $a_{00}$  coefficients following equation (57). Its time evolution is fitted by a 3rd order polynomial (solid line). *Bottom:* the mass accretion history  $\log M(z)/M(z=0)$  for three different class of masses. Masses are expressed in solar masses. Symbol represent the median value of  $\log M(z)/M(z=0)$  within each classes. Lines represent the fitting function suggested by van den Bosch (2002a). Even though the global behavior is reproduced by the fitting functions, the measured accretion rate is systematically smaller. This discrepancy has already been noted by van den Bosch (2002a).

value of  $\Psi(t)$  for three classes of mass was compared to the fit suggested by van den Bosch (2002a) (see Fig. (14)). For the three classes, the agreement with the fitting formula is qualitatively satisfying: the three measurements evolve in the expected manner while their relative positions are the same as the relative positions of the three fits. However our measurements are quantitatively inconsistent with the three curves. At low redshift,  $\Psi(t)$  is systematically larger than the expected value (i.e. the accretion rate is *smaller*). The median mass at  $z=1$  is well recovered even though the two method disagree slightly quantitatively. In other words, our measurements overestimates the accretion at high redshift and underestimates accretion at low redshift and from the measurement procedure through the sphere. At higher redshift, accreted material is assumed to be added to the biggest progenitor, even though it

has not yet reached the central object: its mass is overestimated. Still, this material end up in the most massive progenitor and the final mass is recovered. Note that since specific selection criteria were applied, these halos may not be completely representative of the whole halos population. Finally recall that the *median* value of  $\Psi(t)$  was represented here because of strong outliers while the fitting formula is given for the *average* MAH (extracted from merger trees). A similar discrepancy had already been noticed by van den Bosch (2002a) between the Press-Schechter prediction and the results obtained from numerical simulations. In particular, van den Bosch (2002a) found that the Press-Schechter models tend to underestimate the formation time haloes compared to simulations. Clearly, our measurements seem to confirm this discrepancy. Since the global behavior of MAHs is recovered and since the median mass at  $z = 1$  is recovered, it is concluded that the measure of  $\Phi^M(t)$  through the virial sphere reproduce the accretion history of halos.

### 5.3 Mean kinematics

Let us now turn to the kinematical properties of the flow, while averaging the source over the virial sphere.

#### 5.3.1 probability distribution of the modulus of velocities

Given the source coefficients  $c_{\alpha,m}^m$ , the average velocity distribution  $\langle\varphi(v,t)\rangle$  (defined by equation (49)) is easily computed since it only involves  $\langle c_{\alpha,0}^0(t)\rangle$ . The ensemble average of  $\langle c_{\alpha,0}^0(t)\rangle$  and the related ensemble dispersion  $\sigma(c_{\alpha,0}^0(t)) \equiv \langle (c_{\alpha,0}^0 - \langle c_{\alpha,0}^0 \rangle)^2 \rangle$  are derived by fitting the  $c_{\alpha,0}^0(t)$  distribution by a Gaussian function. From these two quantities, it follows:

$$\langle\varphi(v,t)\rangle = 4\pi v^2 \sum_{\alpha} g_{\alpha}(v) \langle c_{\alpha,0}^0 \rangle, \quad (58)$$

and

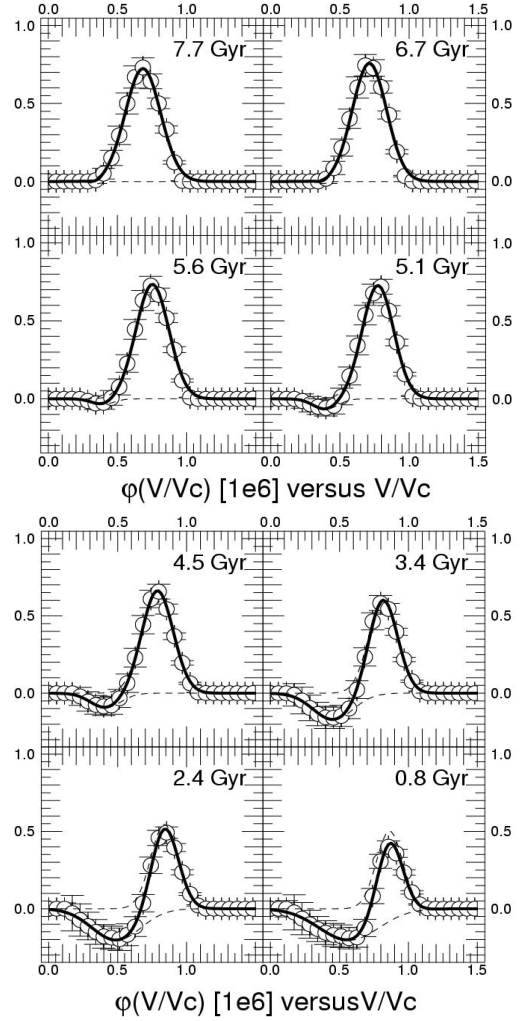
$$\sigma[\varphi(v,t)] = 4\pi v^2 \sqrt{\sum_{\alpha} g_{\alpha}(v)^2 \sigma(c_{\alpha,0}^0)^2}, \quad (59)$$

which are respectively the ensemble average and r.m.s. of the velocity distribution. The time evolution of  $\langle\varphi(v,t)\rangle$  is given in Fig. (15) and (16). Errors on  $\langle\varphi(v,t)\rangle$  are computed as

$$\Delta[\langle\varphi(v,t)\rangle] = 3 \frac{\sigma[\varphi(v,t)]}{\sqrt{N_{\text{halos}}}}. \quad (60)$$

At ‘early times’ ( $t > 5$  Gyr), the distribution is unimodal with a maximum around  $0.7V_c(z=0)$ . No outflows can be detected at any velocity and the infalling dark matter dominates. At later times,  $\langle\varphi(v,t)\rangle$  drops below zero for velocities around  $0.4V_c(z=0)$ . Outflows dominate at ‘low’ velocities. Meanwhile the amplitude of the previous peak decreases and shift to higher velocities. The fraction of infall relative to the total amount of material passing through the sphere drops from 1. to 0.6 between  $t = 8$  Gyr and  $t = 0.8$  Gyr.

This behavior is likely to be due to our measurement at a *fixed* radius,  $R_{200}(z=0)$ . At ‘early times’, this measurement radius is bigger than the actual virial radius of halos. Thus no sign of ‘virialization’ (outflows consecutive to accretion) is detected. Later, the actual  $R_{200}$  gets closer to the measurement radius. Outflows pass through the measurement radius as a sign of internal dynamical reorganization. The fact that accretion intrinsically decreases with



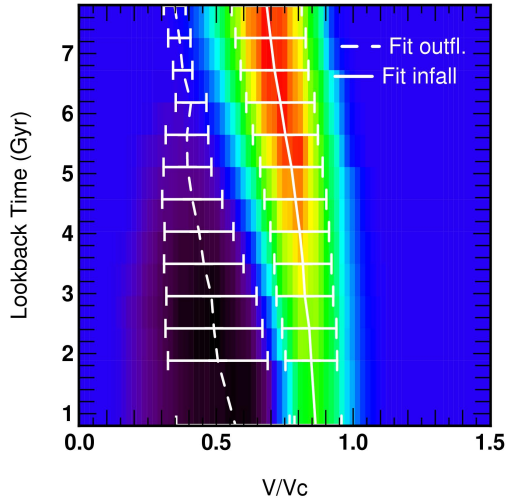
**Figure 15.** The time evolution of the average velocity distribution,  $\langle\varphi(v,t)\rangle$ , defined by equation (49), for  $z < 1$  (symbols). Ages are expressed as look-back times (i.e.  $t = 0$  for  $z = 0$ ). Velocities are given relative to the halo’s circular velocity at  $z = 0$ . Y-axis unit is  $5 \cdot 10^9 M_{\odot}/\text{kpc}^2/\text{Myr}/V_c$ . Error bars stand for  $3 - \sigma$  errors. Here  $\varphi(v)$  is fitted by the sum of two Gaussians with opposite signs (solid line). Each Gaussian’s contribution is also shown (dashed lines).

time would provide another explanation of this trend. This decrease can actually be traced in Fig. (15), (16) and (D1).

The global behavior of  $\langle\varphi(v,t)\rangle$  can be modeled by summing two Gaussians representing the infalling and outflowing components :

$$\langle\varphi(v,t)\rangle = \frac{q_{i,3}(t)}{q_{i,2}(t)\sqrt{2\pi}} \exp\left(-\frac{(v - q_{i,1}(t))^2}{2q_{i,2}(t)^2}\right) + \frac{q_{o,3}(t)}{q_{o,2}(t)\sqrt{2\pi}} \exp\left(-\frac{(v - q_{o,1}(t))^2}{2q_{o,2}(t)^2}\right). \quad (61)$$

Subscripts  $i$  and  $o$  stand for infall and outflow. Note that  $q_{i,3}(t) \geq 0$  and  $q_{o,3}(t) \leq 0$ . The coefficients time evolution is given in Fig. (D1), where  $t$  should be expressed in Gyr and  $\varphi(v,t)$  unit is  $5 \cdot 10^9 M_{\odot}/\text{kpc}^2/\text{Myr}/V_c$ . Examples of fits are shown as solid lines in Fig. (15). Note that all the six coefficients evolve roughly linearly with time (see Fig. (D1)). Their linear fitting parameters are given in table D1. Using equation (61) and the linear parameterization of the Gaussians coefficients,  $\langle\varphi(v,t)\rangle$  is reproduced accurately. The



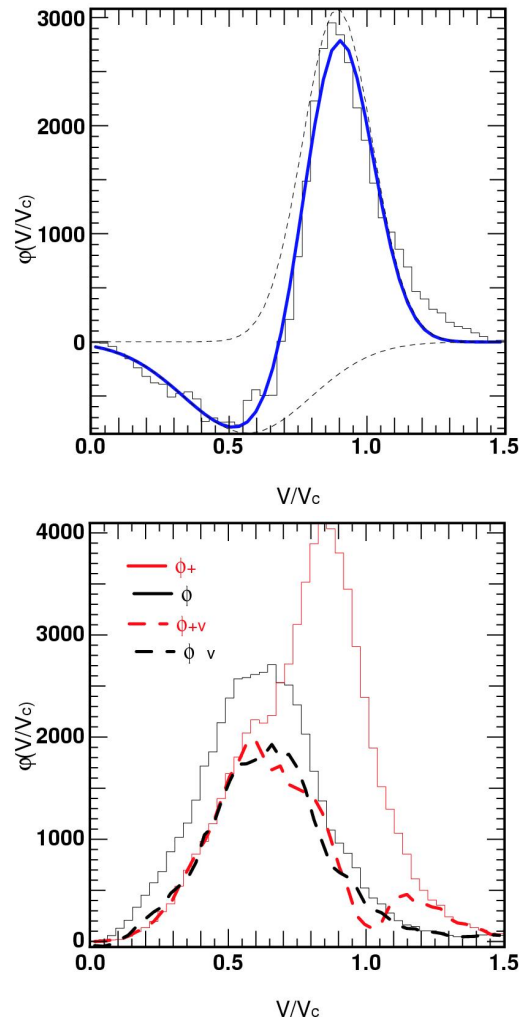
**Figure 16.** The time evolution of the average velocity distribution in the  $t$ - $V/V_c$  plane. Red symbols stand for positive values of the distribution (i.e. inflow) while blue symbols stand for negative ones (i.e. outflows). Each of these components is fitted by a Gaussian function in the  $V/V_c$  space. The time evolutions of the mean of the Gaussians are given by the two lines (solid for inflow, dashed for outflows). The Gaussians r.m.s. are also shown as bars.

only restriction concerns the negative Gaussian’s amplitude ( $q_{o,3}$ ) which should not be greater than 0. Since this condition is not naturally satisfied by a linear fit, it should be set by hand.

The evolution of the relative positions of two Gaussians is given in Fig. (16). For  $t > 5$  Gyr, it is consistent with the ‘no outflow’ hypothesis, the amplitude of the negative Gaussian being close to zero at this epoch. Both Gaussians mean values seem to drift to higher velocities as a function of time. Even though the relative velocity of accreted material is determined by the initial conditions (namely large scale clustering), the velocity of an infalling satellite should partly reflect the properties of the accreting body. A dense massive halo will not accrete like a fluffy light one. In other words, the velocity of infalling material should reflect the actual circular velocity of the accreting body. As a consequence it is expected that accretion velocity drifts with time towards  $V_c(z = 0)$ .

Furthermore, the mean values of both Gaussians evolves roughly linearly over the whole time range with comparable rate of change (see Fig. (D1) and the following discussion). As a consequence, their relative positions remains roughly constant (see Fig. (16)). This indicates that these two components may be physically related, outflows being the consequence of a past accretion. Mamon et al. (2004) mention the existence of a *backsplash* population, rebounding through the virial radius and this population is known to have a different velocity (e.g. Gill et al. (2004)). The outflows detected via our description of the source is consistent with this backsplash component. The difference in velocity may be explained if outflows are representative of an earlier accretion with a velocity typical of earlier times. Also past accreted material is influenced by the halo’s internal dynamics. Its velocity distribution would be ‘reprocessed’ (e.g. via dynamical friction, tidal stripping or phase mixing) to lower velocities as the material exits through the measurement radius.

However, recall that the distribution shown here is a ‘net’ distribution. In other words, it is quite plausible that an outflowing component may be completely canceled by an infalling component which has an exact opposite distribution. This effect is illustrated



**Figure 17.** *Top:* the net velocities distribution (histogram) measured from 300 haloes at  $t=3.4$  Gyr. This distribution is representative of the distribution computed from coefficients and averaged over 15 000 haloes. Velocities are given in circular velocity unit while y-axis units are arbitrary. The two components are fitted by two gaussians (dashed lines). *Bottom:* the separate velocities distribution of accretion (red histogram) and outflows (black histogram). The dashed curves represent the difference between these two distributions and their respective fits shown above. It results in two residual distributions, centered on the same velocity and displaying nearly the same shape. These two residual distributions describe the material which already experienced one passage through the virial sphere.

by Fig. (17) where the velocity distribution of infall and outflows are being shown separately. This distribution has been computed from 300 haloes at  $t=2.3$  Gyr. This distribution is quite representative of the average ones, except a few high velocities events which skew the distribution of the infalling component and which are induced by outliers. If the gaussians fits are removed from these two separate distributions, two almost identical distributions appear for the two components, centered on  $V/V_c \sim 0.6$ . These two identical distributions are related to the virialised component of infall, which already interacted with the inner region of the halo. The overall shape of these two distributions may provide insights on the typical dynamical state in the haloes’ inner regions.

### 5.3.2 Impact parameters and incidence angles

The average distribution of incidence angle,  $\langle \vartheta(\Gamma_1, t) \rangle$  has been computed following the same procedure described above for  $\langle \varphi(v, t) \rangle$ . Defining  $\tilde{c}_{\ell'}(t)$  as:

$$\tilde{c}_{\ell'}(t) = 2\pi\sqrt{4\pi} \sum_{\alpha} c_{\alpha,0,\{\ell',0\}} (\mu_{\alpha}^2 + \sigma^2), \quad (62)$$

yields

$$\langle \vartheta(\Gamma_1, t) \rangle = \sum_{\ell'} Y_{\ell',0}(\mathbf{\Gamma}) \langle \tilde{c}_{\ell'}(t) \rangle, \quad (63)$$

and

$$\sigma(\vartheta(\Gamma_1, t)) = \sqrt{\sum_{\ell'} Y_{\ell',0}(\mathbf{\Gamma})^2 \sigma(\tilde{c}_{\ell'}(t))^2}, \quad (64)$$

where  $\langle \vartheta(\Gamma_1, t) \rangle$  and  $\sigma(\vartheta(\Gamma_1, t))$  are derived by fitting the  $\tilde{c}_{\ell'}(t)$  distribution by a Gaussian function. Errors on  $\langle \vartheta(\Gamma_1, t) \rangle$  are computed similarly to errors on  $\langle \varphi(v, t) \rangle$  (see equation (60)). The time evolution of  $\langle \vartheta(\Gamma_1, t) \rangle$  is shown in Fig. (18). Since the impact parameter and the incidence angle are simply related by  $b/R_{200} = \sin(\Gamma_1)$ ,  $\langle \vartheta(b, t) \rangle$  is also easily computed.

The infall ( $\Gamma_1 > \pi/2$  or the upper branch in  $\langle \vartheta(b, t) \rangle$  diagrams) is clearly mostly radial. The infalling part of the distribution peaks for  $\Gamma_1 \sim \pi$  instead of having a uniform behavior and this trend can be observed for all redshifts below 1. The distribution slightly widens with but remains skewed toward large values of  $\Gamma_1$ . In the  $\langle \vartheta(b, t) \rangle$  representation, the higher branch becomes flatter with time. The outflows ( $\Gamma_1 < \pi/2$  or lower branch in  $\langle \vartheta(b, t) \rangle$  diagrams) are mainly undetectable at early times, as mentioned earlier. As time increases, the outflow contribution becomes stronger and radial orbits ( $\Gamma_1 \sim 0$ ) also appear to be dominant. However the behavior of the ‘outflowing’ part of the  $\langle \vartheta(\Gamma_1, t) \rangle$  distribution is almost linear and does not peak. Tangential orbits cannot be neglected for this component.

The evolution of  $\langle \vartheta(\Gamma_1, t) \rangle$  can be fitted by the following parametrisation:

$$\langle \vartheta(\Gamma_1, t) \rangle = \frac{p_0}{\sqrt{2\pi p_1(t)}} \exp\left(-\frac{(\Gamma_1 - \pi)^2}{2p_1(t)^2}\right) + p_2(t)\Gamma_1 + p_3(t), \quad (65)$$

where  $p_0 = 2 \cdot 10^{-6}$  in our units. The ‘infalling’ part is modeled as a Gaussian while the ‘outflowing’ part is fitted linearly. The time evolution of the three parameters  $p_k(t)$ ,  $k = 1, 2, 3$  can be fitted by a linear evolution and the related linear parameters are given in table D2. The evolution of  $p_1(t)$  confirms that the ‘infalling’ part of the distribution,  $\langle \vartheta(\Gamma_1, t) \rangle$  widens with time.

This result implies that material experiences a circularisation as it interacts with the halo. Consequently, orbits are more tangential as particles *exits* and *re-enter* the halo’s sphere. Such an effect has already been measured by e.g. Gill et al. (2004). Dynamical friction would provide a natural explanation for this evolution of the orbits, but this argument is refuted by e.g. Colpi et al. (1999) or Hashimoto et al. (2003). Gill et al. (2004) mention the secular evolution of halos to explain this circularisation: the time evolution of the potential well induced by the halo would affect the orbits of infalling material and satellites. Other processes such as tidal stripping or satellite-satellite interactions may also modify the orbital parameters of dark matter fluxes. Clearly, the interactions between the infall and the halo drive this circularisation but the detailed process still has to be understood.

This dynamical circularisation could also explain why the ‘outflowing’ part of the  $\langle \vartheta(\Gamma_1) \rangle$  (or  $\langle \vartheta(b) \rangle$ ) is flatter than the in-

falling one: by definition this component interacted with the halo in the past, unlike most of the infall. Finally, the  $\Gamma_1$  or  $b$  representation explicitly separates infall and outflows. It implies that *virialized* particles which pass through the sphere do not ‘cancel’ each other and do contribute to the distributions. Such a ‘relaxed’ material is likely to have a non zero tangential motion, flattening the distributions as its contribution becomes important. Since the actual size of the halo gets closer to the measurement radius as time advances, this component contributes more with time and its flattening effect on the incidence angle (or impact parameter) distributions should increase as well.

Fig. (20) presents the correlation between the velocity amplitude  $v$  and the impact parameter  $b$ , at four different instants and for both the infall and the outflow component. Considering these two components separately, no correlation can be found: the incidence does not depend on the amplitude on first approximation. The only noticeable result comes from the fact that accreted material has systematically a higher velocity than outflows, which confirms the results obtained from the distribution of velocities only. Again, this effect is related to the separate origin of these two fluxes, accretion being dominated by newly accreted material and outflows which were processed by the inner dynamics of haloes.

## 6 TWO POINT STATISTICS

Let us now focus on second order statistics, through the correlation on the virial sphere. The two point correlations are assessed through the angular power spectrum and the angulo-temporal correlation function for both the external potential,  $\psi^e$  and the first moment of the source term, i.e. the flux density of mass,  $\varpi\rho$ .

### 6.1 External potential

#### 6.1.1 Angular power spectrum

The potential’s angular power spectrum  $C_{\ell}^{\psi^e}$  is computed for each halo from the  $\tilde{b}_{\ell,m}$  coefficients (Aubert et al. (2004)):

$$\tilde{b}_{\ell,m} \equiv \sqrt{4\pi} \left( \frac{b_{\ell,m}}{\langle b_{00} \rangle} - \delta_{\ell 0} \frac{b_{0,0}}{\langle b_{00} \rangle} \right), \quad (66)$$

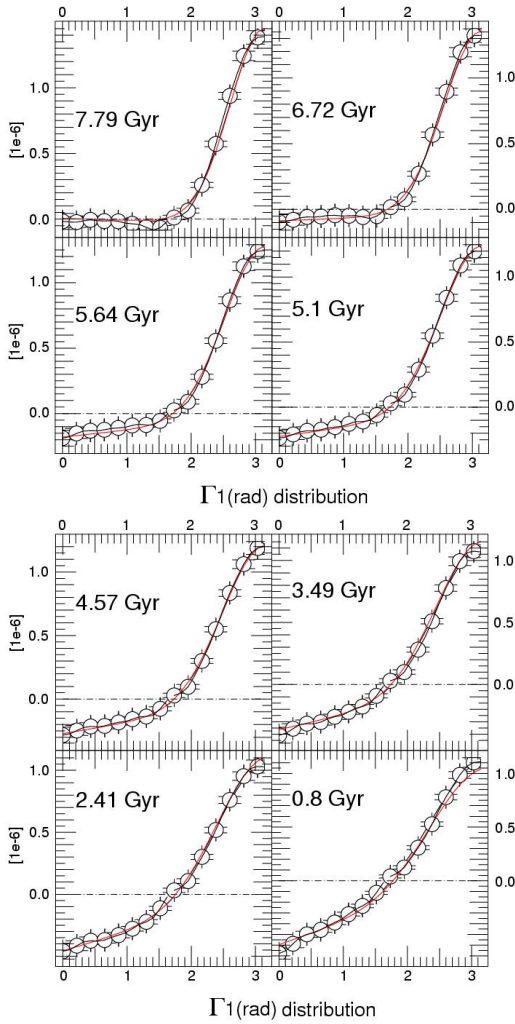
related to the potential contrast :

$$\delta_{[\psi^e]}(\mathbf{\Omega}) \equiv \frac{\psi^e(\mathbf{\Omega}) - \overline{\psi^e}}{\langle \psi^e \rangle} = \sum_{\ell,m} \tilde{b}_{\ell,m} Y_{\ell,m}(\mathbf{\Omega}). \quad (67)$$

The probability distribution of  $C_{\ell}^{\psi^e}(t)$  was weighted as described in appendix A. For each time step and for each harmonic  $\ell$ , the  $C_{\ell}^{\psi^e}$  was fitted by a log-normal distribution (see Fig. C2). Let us define  $\langle\langle C_{\ell}^{\psi^e} \rangle\rangle(t)$  as the mode of the fitting distribution. The time evolution of the external potential’s power spectrum is shown in Fig. 21. Globally the power spectrum is dominated by large scales and is quite insensitive to time evolution. However, two regime may be distinguished. For low order harmonics,  $\langle\langle C_{\ell}^{\psi^e} \rangle\rangle(t)$  remains mostly constant. For smaller scales ( $\ell > 5$ ),  $\langle\langle C_{\ell}^{\psi^e} \rangle\rangle(t)$  increases along time. As a consequence the power spectrum’s amplitude does not change but its shape evolves while smaller scales become more important relative to larger scales.

These two regimes reflect the twofold nature of tidal interactions of a halo with its environment. Small angular variations of the potential relate to small spatial scales and presumably track the presence of objects which are getting closer or going through the

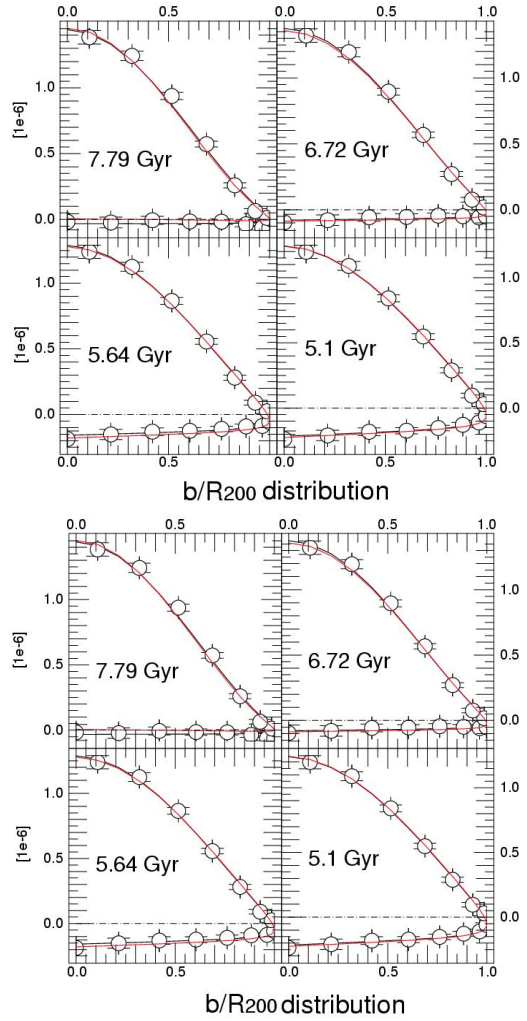




**Figure 18.** The time evolution (symbols) of the distribution of the average incidence angle  $\Gamma_1$  ( $\langle \vartheta(\Gamma_1) \rangle$ ), defined by equation (47). Ages are expressed as look-back time. Bars stand for  $3\sigma$  errors. Y-axis unit is  $5 \cdot 10^9 M_\odot / \text{kpc}^2 / \text{Myr}$ . Outflows are counted negatively, leading to negative values of  $\langle \vartheta(\Gamma_1) \rangle$  for  $\Gamma_1 < \pi/2$ . The result of the model described in equation (65) is also shown (red line).

virial sphere. Since small scales contribution increases, it suggests that these objects tend to get smaller with time. It would be consistent with the global decrease of the accretion rate, as long as the merger rate does not increase strongly during this epoch. However, the rise of small scales may also be related to an increasing contribution of weak and poorly resolved accretion events. In such a case, the isolated particles contribution to the potential should be measured. This possibility is investigated in section 6.2.

Meanwhile, large scale potential's fluctuations ( $\ell \leq 4$ ) may reflect the 'cosmic tidal field' resulting from the distribution of matter around the halo on scales larger than the halo's radius. The amplitude of such a tidal field should remain fairly constant, as indeed measured. Furthermore, the peripheral distribution of matter is not spherically distributed but is rather elongated along some direction: haloes tend to be triaxial with their ellipsoid aligned with the surrounding distribution of satellites. The intersect of an elongated distribution of matter with the virial sphere would induce a quadrupolar component, as detected in our measurements. These two effects cannot be easily disentangled, since they actually are



**Figure 19.** The time evolution (symbols) of the impact parameter  $b$  distribution ( $\langle \vartheta(b, t) \rangle$ ), defined by equation (47). Ages are expressed as look-back time. Bars stand for  $3\sigma$  errors. Y-axis unit is  $5 \cdot 10^9 M_\odot / \text{kpc}^2 / \text{Myr}$ . The lower (resp. higher) branch is the  $\langle \vartheta(b, t) \rangle$  distribution for outflows (resp. infall). The result of the model described in equation (65) is also shown (red line).

two sides of the same effect. Large scale distribution of matter is responsible of both the 'cosmic tidal field' and the halo triaxiality (via the distribution of satellites). In other words, it is not clear whether the large scale behavior of  $\langle C_\ell^{\psi^e} \rangle(t)$  reflects the tidal field or the halo's reaction to this tidal field.

### 6.1.2 Angulo-temporal correlation

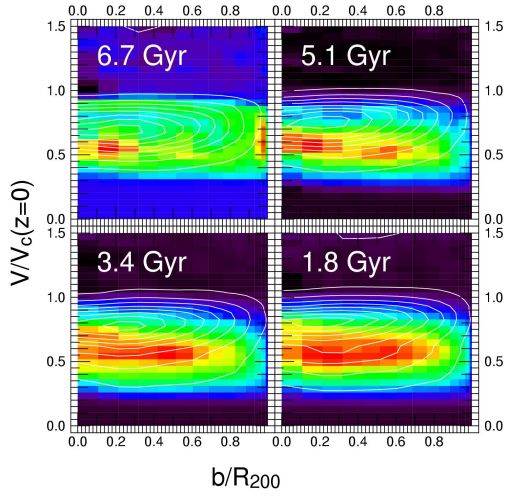
**6.1.2.1 Correlations and coherence time** To investigate further these two regime of tidal interactions, let us compute the angulo-temporal correlation function of the external potential contrast, defined as

$$\langle\langle w^e(\theta, t, t + \Delta t) \rangle\rangle = \langle\langle \delta_{[\psi^e]}(\Omega, t) \delta_{[\psi^e]}(\Omega + \Delta\Omega, t + \Delta t) \rangle\rangle, \quad (68)$$

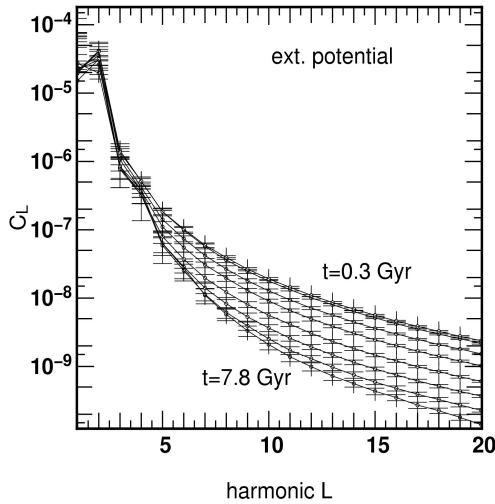
which is related to  $T_\ell^{\psi^e}(t, t + \Delta t)$  coefficients by:

$$w^e(\theta, t, t + \Delta t) = \sum_\ell T_\ell^{\psi^e}(t, t + \Delta t) (2\ell + 1) P_\ell(\cos(\theta)), \quad (69)$$

where



**Figure 20.** *Top:* the distribution,  $\langle \varphi(b, v) \rangle$ , of particles in the  $(b, v)$  subspace at lookback time  $t = 1.8, 3.4, 5.1$  and  $6.7$  Gyr; the infall (contour plot) and outflow (density plot) are represented separately. Beyond the bimodal feature, no residual correlation appears.



**Figure 21.** The angular power spectrum of the external gravitational potential  $\psi^e(\Omega, t)$ . Symbols represent the *mode* of the  $C_\ell^{\psi^e}$  distribution for each harmonic  $\ell$  and each time step. Times are lookback times. Bars stand for  $3 - \sigma$  errors on the mode value. The large scales contribution remains constant with time while the small scales contribution smoothly increases with time. The bump for the  $\langle C_2 \rangle$  component indicates a strong quadrupolar configuration for  $\psi^e(\Omega, t)$ .

$$T_\ell^{\psi^e}(t, t + \Delta t) \equiv \frac{1}{4\pi} \frac{1}{2\ell + 1} \sum_m \tilde{b}_{\ell m}(t) \tilde{b}_{\ell m}^*(t + \Delta t). \quad (70)$$

Here,  $\theta$  stand for the angular distance between two points on the sphere located at  $\Omega$  and  $\Omega + \Delta\Omega$ . The  $T_\ell^{\psi^e}(t, t + \Delta t)$  coefficients were computed for each halo and each pair of time step, for each harmonic. The  $T_\ell^{\psi^e}(t, t + \Delta t)$  distributions were fitted by a log normal distribution and  $\langle T_\ell^{\psi^e}(t, t + \Delta t) \rangle$  was deduced from it. The corresponding  $\langle w^e(\theta, t, t + \Delta t) \rangle$  are shown in figure Fig. (22).

For large angular scales ( $> 45$  degrees), isocontours remain open during the whole time range. Large scales have a long coherence time ( $\sim 5$  Gyrs) and are consistent with a ‘cosmic tidal field’

resulting from the large scale distribution of matter. The latter is not expected to evolve significantly with time at our redshifts and the halo’s triaxiality is also a fairly constant feature. The innermost isocontours are closed around the measurement time  $t_1$ . Small angular scales ( $< 45$  degrees) have shorter coherence time ( $\sim 1.5$  Gyr). This is consistent with a contribution to the potential due to objects where satellites pass by or dive into the halo and apply a tidal field only for a short period.

This difference between large and small scales can also be investigated through the time matrices of the  $T_\ell^{\psi^e}(t, t + \Delta t)$  coefficients (see Fig. (23)). The diagonal terms describe the time evolution of the angular power spectrum,  $T_\ell^{\psi^e}(t, t) = C_\ell^{\psi^e}(t)$ . A smooth (resp. peaked) distribution of values around the diagonal indicates a long (resp. short) coherence time. Clearly different scales have different characteristic time scales. The non-diagonal elements of the quadrupole matrix ( $\ell = 2$ ) decrease slowly with the distance to the diagonal while the  $T_{20}$  matrix is almost diagonal. Not surprisingly, the smaller the angular scale, the smaller the coherence time: a small 3D object passing through the sphere is likely to have a small angular size on the sphere.

For a given  $\ell$  and a given  $t$ , the correlations coefficients  $T_\ell^{\psi^e}$  can be fitted by a Lorentzian function defined by

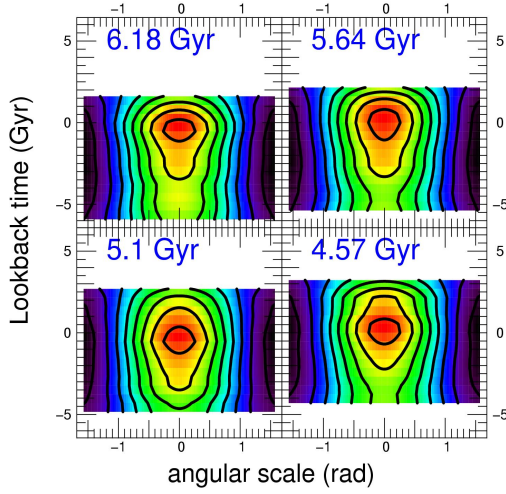
$$T_\ell(t, t + \Delta t) = \frac{q_3^{\text{Te}}(t)}{2/\pi} \frac{q_2^{\text{Te}}(t)}{(\Delta t - q_1^{\text{Te}}(t))^2 + (q_2^{\text{Te}}(t)/2)^2} + q_4^{\text{Te}}(t). \quad (71)$$

where the characteristic time scale,  $\Delta T_{T_\ell^e}$ , is given by  $q_2^{\text{Te}}(t)$  and the reference time  $t$  is equal to  $q_1^{\text{Te}}(t)$ . Example of fits are shown in Fig. (24).

For example, the time evolution of  $\Delta T_{T_\ell^e} = q_2^{\text{Te}}(t)$  is given in Fig. (24) for different  $\ell$  values. Given the error bars, the characteristic time scales are constant over time (except for the  $\ell = 4$  mode). In the prospect of the regeneration of the potential, the stationarity hypothesis can then be considered as valid for most of the angular scales. Meanwhile, the  $\ell = 4$  potential fluctuations display a decreasing  $\Delta T_{T_\ell^e}$  with time. The same effect exists at a  $1\sigma$  level for  $\ell = 5$ . One interpretation would be that satellites achieve higher velocities along time: for a given typical size a faster satellite would spend less time to be accreted and the associated potential would be detected on a smaller time scale. This picture is supported by the results shown in 5.3.1, where the mean velocity of infalling material increases along time. Another possibility would be that  $\ell = 4$  fluctuations had a longer radial extent in the past. Since there is no reasons for potential fluctuations to have such a property, one could imagine *successive* potential fluctuations which overlapped, leading to an apparent longer radial extent. This possibility is further investigated in the following paragraphs, by comparing the coherence time variation of the potential fluctuations to the evolution of the infall’s typical velocity .

### 6.1.2.2 Correlations without the dipole and the quadrupole

In order to focus on the coherence time of small angular scales, the correlation function  $\langle w^e(\theta, t, \Delta t) \rangle$  was also computed without the dipole ( $\ell = 1$ ) and the quadrupole ( $\ell = 2$ ) component of the potential (see also equation (69)). The angulo-temporal correlation function is shown in Fig. (25). Again, the isocontours of the correlation function are closed around  $\Delta t = 0$ . This shows that the potential on the sphere have a finite coherence time. In contrast to coherence times measured on the  $T_\ell^{\psi^e}$  coefficients, all the angular scales are mixed and the typical time scales are those of structures as they are ‘really’ seen from a halocentric point-of view where ‘potential blobs’ appear and disappear on the sphere. To evaluate the related



**Figure 22.** The angulo-temporal correlation function,  $w^e(\theta, \Delta t) = \langle \delta_{[\psi^e]}(\Omega, t) \delta_{[\psi^e]}(\Omega + \Delta\Omega, t + \Delta t) \rangle$ . Blue (resp. red) colors stand for low (resp. high) values of the correlation. Isocontours are also shown. Large angular scale isocontours ( $\theta \sim \pi/2$ ) have large temporal extent, due to the quadrupole dominance over the potential seen on the virial sphere.

typical time scale  $\Delta T_{\psi^e}$ ,  $\langle w^e(\Delta\Omega, t, \Delta t) \rangle$  was fitted with a 2D function for different values of  $t$ . The model used is given by :

$$\langle w^e(\theta, t, \Delta t) \rangle = q_6^{\text{we}}(t) + \frac{q_5^{\text{we}}(t)}{2\pi} \frac{q_4^{\text{we}}(t)}{(\Delta t - q_3^{\text{we}}(t))^2 + (\frac{q_4^{\text{we}}(t)}{2})^2} \times \frac{\sin(2\pi(\Delta\Omega - q_1^{\text{we}}(t))/q_2^{\text{we}}(t))}{\Delta\Omega}, \quad (72)$$

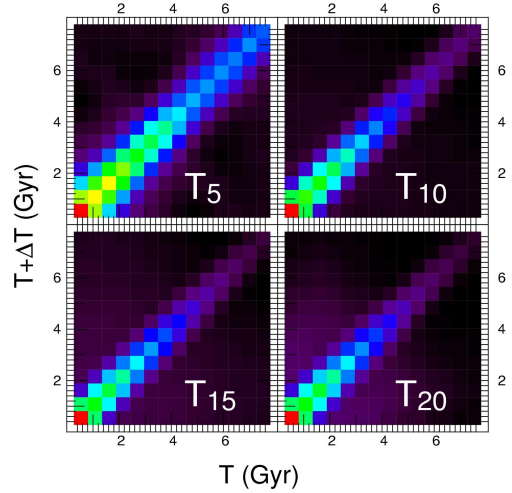
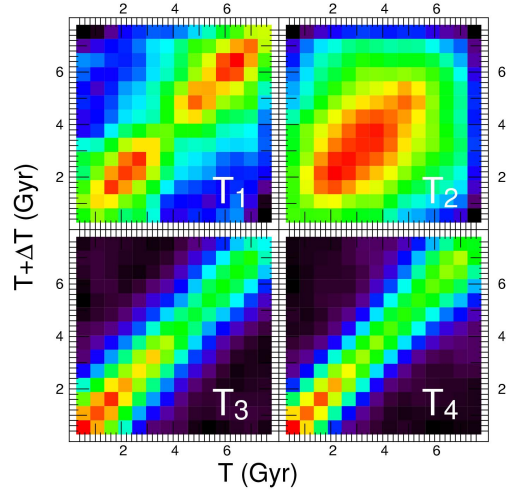
where the angular dependence is fitted by a cardinal sine function while the time dependence is fitted by a Lorentzian function. Examples of 2D fits are shown in Fig. (25). The correlation function was also computed using only harmonics with  $\ell \geq 4, 5, 6, 7$  and the same fitting procedure is applied. The evolution of the resulting characteristic time scales  $\Delta T_{w^e} = q_4^{\text{we}}(t)$  is shown in Fig. (26). Bars stand for  $3\sigma$  fitting errors.

Note that  $\Delta T_{w^e}$  tends to decrease with time for every truncation order. The  $\ell \geq 3$  and  $\ell \geq 4$  correlation function displays a rise of  $\Delta T_{w^e}$  before it drops to lower values. Furthermore  $\Delta T_{w^e}$  tend to decrease with  $\ell_{\text{min}}$ , suggesting that the  $\ell_{\text{min}}$  contribution dominates each  $w^e$  reconstruction. Correlation functions with  $\ell_{\text{min}} \geq 5$  show marginal  $\Delta T_{w^e}$  variation but recall that our time resolution is 0.53 Gyr, hence any fluctuations on smaller scales should be taken in caution. Still, the 0.81 Gyr variation observed for  $\ell \geq 3$  between  $t = 5.1$  Gyrs and  $t = 0.8$  Gyr is significant and so is the variation observed for  $\ell \geq 4$  (1.3 Gyr).

**6.1.2.3 A longer coherence length** As mentioned above, the variation of the characteristic time scale can be explained by the measured increase in mean velocity. Conversely, the decrease of coherence time may be the consequence of smaller potential blobs as time passes: a ‘large’ (three dimensional) potential takes longer to disappear than a smaller one. One crude approximation could be :

$$\frac{L_1}{L_2} = \frac{V_1 \Delta T_1}{V_2 \Delta T_2}. \quad (73)$$

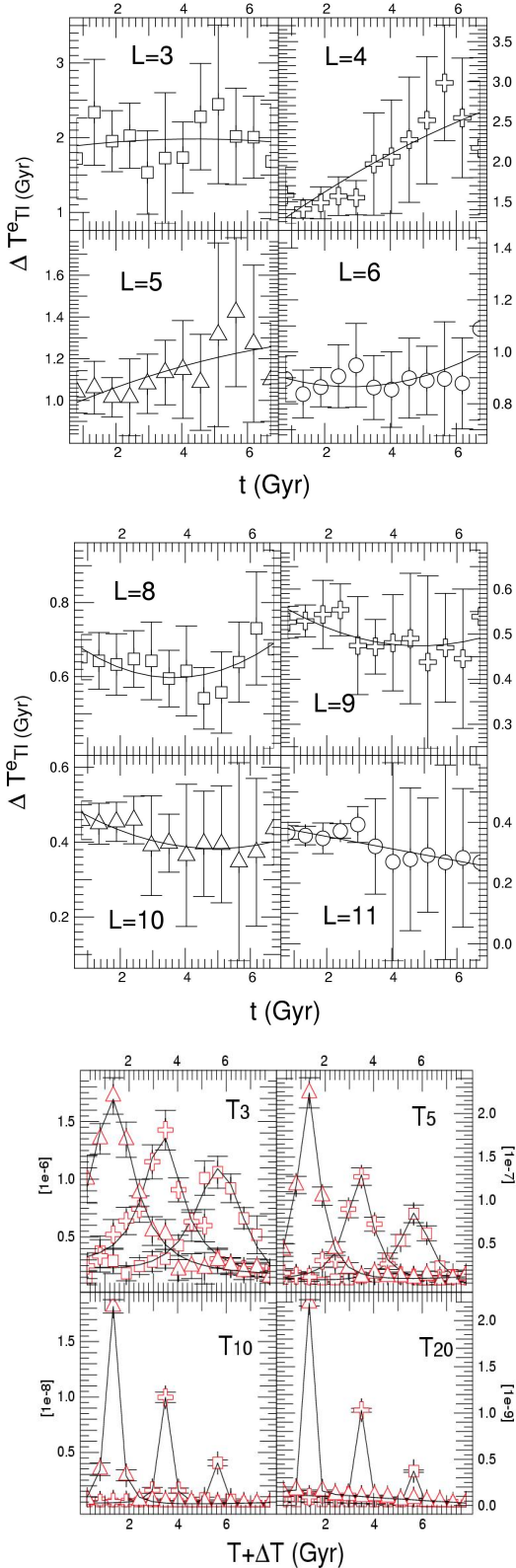
where  $L$  is the radial size of the potential blob,  $V$  its radial velocity and  $\Delta T$  its coherence time. It is assumed that the  $\ell \geq 4$  truncation is representative of the potential due to infalling objects, i.e.



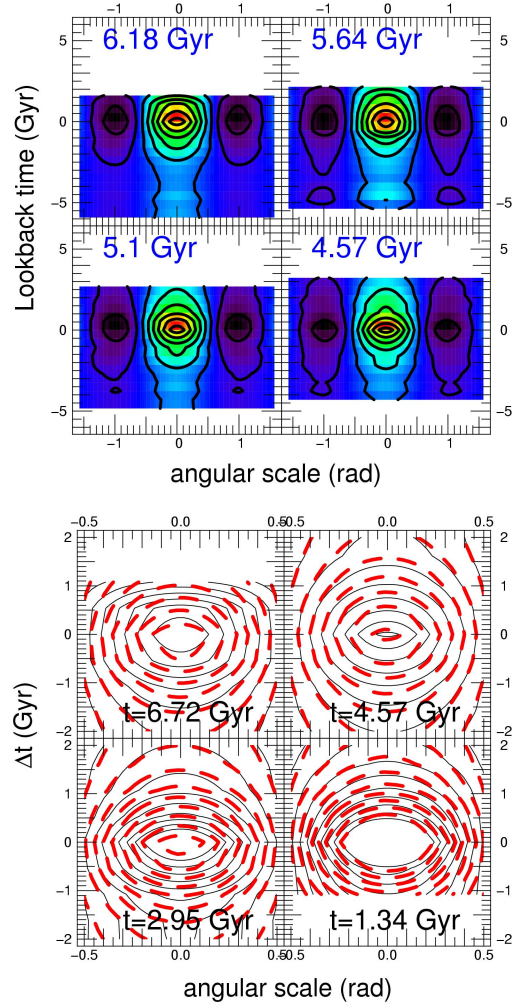
**Figure 23.** The time matrices of the  $T_\ell^{\psi^e}(t, t + \Delta t)$  coefficients. Blue (resp. red) colors stand for low (resp. high) values of the coefficients. The diagonal terms are equal to the angular power spectrum, i.e.  $T_\ell^{\psi^e}(t, t) = C_\ell^{\psi^e}(t)$ . As can be seen from Fig. (21) fluctuations observed for  $T_1$  are within the error bars. A smooth (resp. sharp) decrease of  $T_\ell^{\psi^e}(t, t + \Delta t)$  with the distance to the diagonal imply a long (resp. short) coherence time. Here, coherence time decreases with angular scale.

$\Delta T_1 / \Delta T_2 \sim 1.95$ . Let us also consider that the radial velocity variation is equal to the one measured for the mean velocity of infall ( see section 5.3.1):  $V_1 / V_2 = 0.77$ . Following equation (73), it suggests that  $L_1 \sim 1.5 L_2$ , i.e. the radial size decreases with time. The same calculation with  $\ell \geq 3$  leads to  $L_1 \sim 1.1 L_2$  : the results remain qualitatively the same. In other words, the *coherence length* was longer in the past and the velocity variation cannot explain the variation of coherence time. The only other way to explain a longer coherence length involves potential blobs falling successively through the sphere, coming from roughly the same direction. To induce a decreasing coherence time, these blobs would have to be either bigger before or either more numerous. Such a crude picture is coherent with the measured decrease of accretion with time and the anisotropic nature of accretion by haloes ( see e.g. Knebe et al. (2004), Aubert et al. (2004), Zentner et al. (2005)).





**Figure 24.** *Top and Middle:* the time evolution of the characteristic time scale  $\Delta T_{T_\ell}^{\psi^e}$ , obtained by fitting  $T_\ell^{\psi^e}(t, t + \Delta t)$  with equation (71). Symbols are the measurements while bars stand for  $3\sigma$  fitting error bars. The second order fit of the time evolution of each  $T_\ell^{\psi^e}$  is also shown. Except for the  $\ell = 4$  and (marginally) for the  $\ell = 5$  modes, no time evolution is observed. The time resolution is 0.53 Gyr. *Bottom:* examples of  $T_\ell^{\psi^e}(t, t + \Delta t)$  fitted by Lorentzian functions.



**Figure 25.** *Top:* the angular-temporal correlation function,  $w^e(\theta, \Delta t) = \langle \delta_{[\psi^e]}(\Omega, t) \delta_{[\psi^e]}(\Omega + \Delta\Omega, t + \Delta t) \rangle$ . The dipole ( $\ell = 1$ ) and the quadrupole ( $\ell = 2$ ) components were removed. Blue (resp. red) colors stand for low (resp. high) values of the correlation. Isocontours are also shown. The main axes of the ‘ellipses’ centered on  $(\Delta\Omega = 0, \Delta t = 0)$  give indications on the characteristic time and angular scales of  $\psi^e(\Omega, t)$ . *Bottom:* comparison between the measured 2D correlation function (*solid lines*) and the fit obtained using the equation (72) (*dashed lines*).

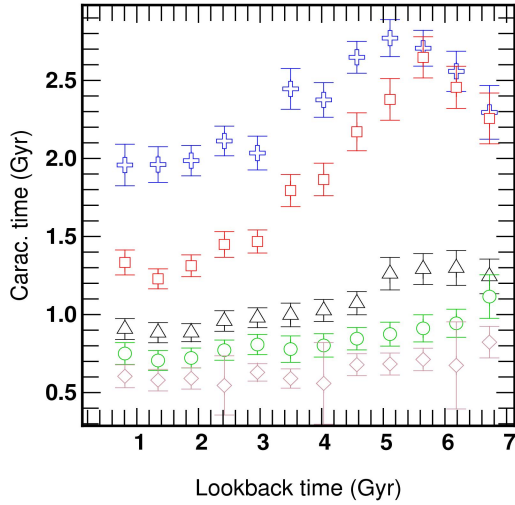
## 6.2 Flux density of mass : $\varpi_\rho \equiv \rho v_r$

The mode  $\langle\langle C_\ell^{\varpi_\rho} \rangle\rangle$  of the distribution of the  $\varpi_\rho$  angular power spectrum is computed using equations (42)-(43). In order to deal with adimensional quantities, the reduced harmonic coefficients,  $\tilde{a}_{\ell,m}$ , are defined as:

$$\tilde{a}_{\ell,m} \equiv \sqrt{4\pi} \left( \frac{a_{\ell,m}}{\langle a_{00} \rangle} - \delta_{\ell 0} \frac{a_{0,0}}{\langle a_{00} \rangle} \right). \quad (74)$$

The accretion contrast,  $\delta_{[\varpi_\rho]}$ , and the  $\tilde{a}_{\ell,m}$  coefficients are linked by:

$$\delta_{[\varpi_\rho]}(\Omega) \equiv \frac{\varpi_\rho(\Omega) - \overline{\varpi_\rho}}{\langle \varpi_\rho \rangle} = \sum_{\ell,m} \tilde{a}_{\ell,m} Y_{\ell,m}(\Omega). \quad (75)$$



**Figure 26.** The cosmic evolution of the potential’s coherence time. This characteristic time scale is obtained by fitting the 2D correlation function  $w^e(\theta, \Delta t)$  with the function given in equation (72). The correlation function is computed using harmonics coefficients with  $\ell \geq 3$  (crosses),  $\ell \geq 4$  (squares),  $\ell \geq 5$  (triangle),  $\ell \geq 6$  (circle) and  $\ell \geq 7$  (diamonds). Bars stand for  $3\sigma$  fitting errors. The time resolution is 0.53 Gyr.

### 6.2.1 Angular power spectrum

Given  $\langle a_{00} \rangle(t)$ , the angular power spectrum  $C_\ell^{\varpi\rho}(t)$  is computed for each halo. At each time step and for each harmonic order  $\ell$ , the  $C_\ell^{\varpi\rho}(t)$  distribution was fitted by a log-normal distribution (see Fig. C4). The probability distribution of  $C_\ell^{\varpi\rho}(t)$  is weighted as described in appendix A.

The evolution of  $\langle C_\ell^{\varpi\rho}(t) \rangle$  with time is shown in Fig. (27). The shape of  $\langle C_\ell^{\varpi\rho}(t) \rangle$  remains mostly the same with time and is fitted by a simple function :

$$\langle C_\ell^{\varpi\rho} \rangle(t) = q_1^{\varpi\rho}(t) + \frac{q_2^{\varpi\rho}(t)}{(\ell + q_3^{\varpi\rho}(t))^2}. \quad (76)$$

The time evolutions of  $q_1^{\varpi\rho}$ ,  $q_2^{\varpi\rho}$  and  $q_3^{\varpi\rho}$  are shown in Fig. (D2) and can be fitted by decreasing exponential:

$$q^{\varpi\rho}(t) = h + k \exp\left(-\frac{t}{u^2}\right). \quad (77)$$

Only the dipole ( $\ell = 1$ ) harmonic does not fit with the previous functional form and is systematically lower than contribution of the other harmonics. If particles velocities were measured in an absolute referential, the dipole strength would reflect the halo’s motion in the surrounding matter. Also strong mergers may cover a 180 degrees angle on the sphere and would contribute to the dipole. Since velocities are measured in the halo’s rest frame and strong mergers are excluded, the dipole strength is substantially lowered, as is measured.

The values for  $h, k, u$  are given in table D3. The offset  $q_1^{\varpi\rho}$  of  $\langle C_\ell^{\varpi\rho}(t) \rangle$  increases with time. From equation (74), one can see that  $\langle C_\ell^{\varpi\rho}(t) \rangle$  is proportional to the square of the accretion contrast. If the power spectrum experiences a global shift toward higher values with time it implies that the accretion contrast increases with time. Since the average velocity does not vary strongly with time, this suggests that we actually see objects getting denser with time. This effect is similar to the global increase of the 3D power spectrum  $P(k)$  with time due to density growth. The  $q_2^{\varpi\rho}$  coefficient evolves as  $q_1^{\varpi\rho}$ . This illustrates the fact that the amplitude of

$\langle C_\ell^{\varpi\rho}(t) \rangle$  remains mainly constant. The  $q_3^{\varpi\rho}$  coefficient should be seen as a typical scale and varies slightly from  $q_3^{\varpi\rho} = 6$  at  $z = 1$  to  $q_3^{\varpi\rho} = 11$  at  $z = 0.1$ . The  $\langle C_\ell^{\varpi\rho}(t) \rangle$  becomes marginally ‘flatter’ as time passes, implying that small scales contribute more to the spatial distribution of  $\varpi_\rho(\Omega, t)$ , consistently with the evolution of  $\langle C_\ell(t)^{\psi^e} \rangle$ . The flat power spectrum measured for  $\varpi_\rho$  on small scales suggests that isolated particles contribute significantly and increasingly with time. In other words, the accretion becomes low enough to be poorly resolved in terms of particles.

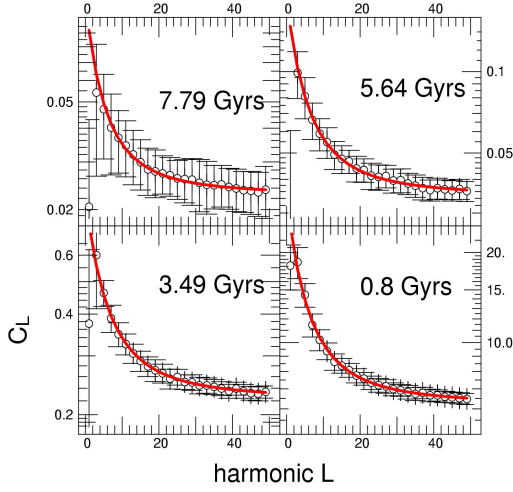
### 6.2.2 Resolution in mass and particle number

In order to assess these environments/resolution effects,  $\langle C_\ell^{\varpi\rho}(t) \rangle$  was computed for three different of masses (see Fig. 28) at a look-back time of 800 Myrs. For the heaviest halos, the power spectrum is peaked toward low  $\ell$  values. The contribution of large scales is quite important. For smaller masses the power spectrum gets flatter and all scales almost contribute equally for the lightest class of mass. Recall that the harmonic decomposition of a Dirac function leads to  $C_\ell = \text{constant}$ . Thus a flat power spectrum indicates that isolated particles contributes significantly to the distribution of matter on the sphere. The relative behavior of the three  $\langle C_\ell^{\varpi\rho}(t) \rangle$  confirms that larger halos still experience important mergers (i.e. on large scales) while small ones are in quiet environments at our simulation resolution. We also investigated the effect of the mass resolution on the angular structure of accretion with two smaller sets of simulations : the first one consists in 10 simulations with  $256^3$  particles in 50 Mpc/h boxes and the second in 5 simulations with the  $256^3$  particles in 20 Mpc/h boxes. The related  $\langle C_\ell^{\varpi\rho} \rangle$  measured at a lookback time of 800 Myrs are shown in Fig. 29. Here 1532 and 545 haloes satisfying the conditions described in 4.2 were detected in these two additional sets of simulations. For clarity,  $1\sigma$  error bars are shown for the two high resolution measurements while the ‘larger statistics’ power spectrum is still represented with  $3\sigma$  bars. For large scales ( $\ell < 10$ ), the three power spectra are consistent, thus suggesting that convergence was achieved there. On smaller scales, the two higher resolution spectra differ significantly from that measured using the other set of simulations (50 Mpc/128<sup>3</sup> particles) : high  $\ell$  hold significantly less power. This confirms that the lack of resolution tend to overestimates the importance of small scales and implies that the study of  $\varpi_\rho$  requires simulations at higher resolution in order to understand e.g. the detailed statistics of small infalling objects. Interestingly, the two higher resolution simulations have identical  $\langle C_\ell^{\varpi\rho} \rangle$ , given the admittedly large error bars. This suggests that statistical convergence at scales  $\ell < 50$  does not requires extremely resolved simulations and simulation boxes with a mass resolution only 8-10 times greater than those used in this paper should suffice.

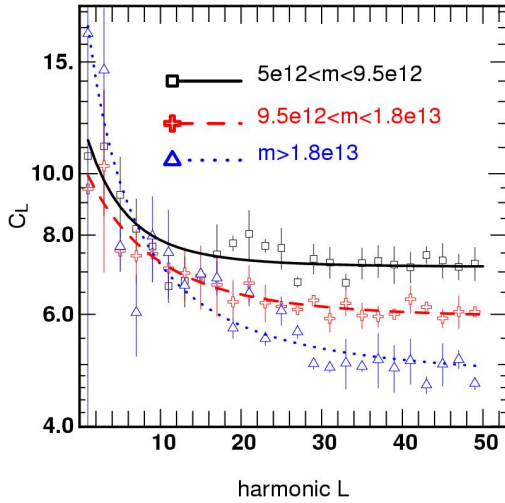
Finally, it clearly appears from Fig. (27) and Fig. (D2) that the angulo-temporal correlation function related to  $\langle C_\ell^{\varpi\rho}(t) \rangle$  are dominated by the overall shift of the angular power spectrum towards higher values and consequently no coherence time should be detectable. It is shown in appendix E how the evolution of  $\langle C_\ell^{\varpi\rho}(t) \rangle$  is related to mass biases and possible resolution effects. The previous measurements on the potential were not sensitive to these effects because of the smoother nature of the field.

In appendix E, the measured secular evolution is avoided by using an alternative definition of  $\langle C_\ell^{\varpi\rho}(t) \rangle$ . It is found that the angular power spectrum of  $\varpi_\rho$  can be fitted by a simple power-law (see Fig. E1) at every time :

$$\langle C_\ell^{\varpi\rho}(t) \rangle \sim \ell^{-1.15}. \quad (78)$$

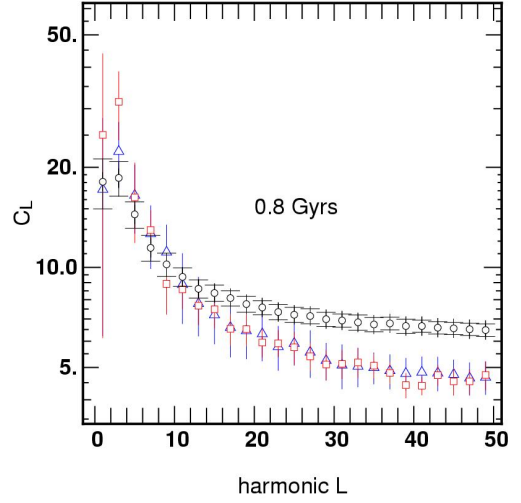


**Figure 27.** The average angular power spectrum,  $\langle C_\ell^{\varpi\rho} \rangle(t)$ , at  $t = 0.8, 3.5, 5.7, 7.8$  Gyr (symbols).  $\langle C_\ell^{\varpi\rho} \rangle(t)$  is taken as the mode of the log-normal function used to fit the  $C_\ell$  distribution. Bars stand for  $3\sigma$  errors. For a given  $\ell$  the corresponding angular scale is  $\pi/\ell$ .  $\langle C_\ell^{\varpi\rho} \rangle(t)$  may be fitted by a generic model given by equation (76) (solid line).



**Figure 28.** The average angular power spectrum,  $\langle C_\ell^{\varpi\rho} \rangle(t)$ , at  $t = 0.8$  Gyr (symbols) for three different class of masses.  $\langle C_\ell^{\varpi\rho} \rangle(t)$  is taken as the mode of the log-normal function used to fit the  $C_\ell^{\varpi\rho}$  distribution. Bars stand for  $3\sigma$  errors. Mass are expressed in solar masses. For a given  $\ell$  the corresponding angular scale is  $\pi/\ell$ . The three measurements are fitted by equation (76) (solid line). The power spectrum gets flatter for small halos. Accretion by small halos is dominated by small objects or even isolated particles.

The corresponding angulo-temporal correlation function is given in Fig. E3. As expected, there is a shorter coherence time for the  $\varpi_\rho$  field than for the potential, because of the ‘sharper’ nature of the former. Overall, these results strongly suggest that class of mass and resolution biases should be systematically investigated beyond what was shown here.



**Figure 29.** The average angular power spectrum,  $\langle C_\ell^{\varpi\rho} \rangle(t)$ , at  $t = 0.8$  Gyr for three different mass resolution in the simulations.  $\langle C_\ell^{\varpi\rho} \rangle(t)$  is taken as the mode of the log-normal function used to fit the  $C_\ell^{\varpi\rho}$  distribution. For a given  $\ell$  the corresponding angular scale is  $\pi/\ell$ . Circles stand for the measurements performed for the main set of simulations (50 Mpc/h,  $128^3$  particles) while error bars stand for  $3 - \sigma$  errors. Square and triangles stand for measurements performed on simulation with higher mass resolution, resp. 50 Mpc/h- $256^3$  particles (1532 haloes analysed) and 20 Mpc/h- $256^3$  particles (545 haloes analysed). In these two cases, error bars stand for  $1\sigma$  errors.

## 7 A SCENARIO FOR THE ACCRETION OF A TYPICAL HALO AT $Z \leq 1$

Let us draw a summary of the previous results, in order to get a synthetic picture of the flux properties at the virial radius. A typical halo in our sample has a mass of  $10^{13}M_\odot$  and a radius  $R_{200} \sim 500\text{kpc}$  at  $z=0$ . It is embedded in a quasi-stationary gravitational potential,  $\psi^e$ . Such a potential is highly quadrupolar and is likely to be induced by the large scale distribution of matter around the halo. The halo accretes material between  $z = 1$  and  $z = 0$  at a rate which declines with time. At high redshift, only accretion is detected at  $R_{200}$ . It is mainly radial and occurs at a velocity close to 75% times the circular velocity. As time advances, accretion of new material decreases, while outflows become significant. Outflows occur at lower velocities and on more circular orbits. A fraction of the outflowing component is due to a backsplash population made of material which already passed through the virial sphere. Another fraction of outflows corresponds to ‘virialized’ material of the halo which goes further than  $R_{200}$  and is being ‘cancelled’ by its infalling counterpart. The clustering on the sphere of the gravitational potential drifts toward smaller scales, while the clustering of matter follows marginally the same trend at our level of resolution. It reflects the increasing contribution with time of weak/diffuse accretion, poorly sampled at our resolution. In parallel, the coherence time of potential fluctuations is found to be decreasing with time by the halocentric observer. This decrease may be related with the accretion of satellites, where objects were numerous enough to ‘overlap’ in the past, which implies that accretion occurs mainly in the same direction on the virial sphere.

This scenario seems consistent with most of the past studies made on the subject using the full 3D information contained in simulations. The decline of accretion rate has been already measured by e.g. van den Bosch (2002b) even though our measurements are in

a slight quantitative disagreement. The ‘rebound’ of matter through the Virial sphere has already been measured by e.g. Mamon et al. (2004) or Gill et al. (2004). Furthermore, the velocity bimodality is recovered together with the circularisation of orbits measured by Gill et al. (2004) in high resolution simulations. Finally, the variation of the potential’s coherence time is found to be related to the anisotropy of accretion, already demonstrated by e.g. Knebe et al. (2004), Aubert et al. (2004) or Zentner et al. (2005).

## 8 SUMMARY & DISCUSSION

In this paper, the second in the series of three, presented measurements of the detailed statistical properties of dark matter flows on small scales ( $\leq 500$  kpc) in the near environment of haloes using a large set of  $\Lambda$ CDM cosmological simulations. The purpose of this investigation was twofold: (i) characterize statistically (via one and two points statistics) the detailed (angular and kinematic) incoming fluxes of dark matter entering the virial sphere of a biased (described in Section 4.2) sample of halos undergoing minor mergers for the broader interest of astronomers concerned with the environment of galaxies; (ii) compute the first two moments of the linear coefficients,  $c_n$ , (resp.  $b_n$ ) of the source term (resp. external potential) entering equations (13)-(14) (paper I).

We concentrated on flows at the haloes’ virial radius while describing the infalling matter via fluxes densities through a spherical shell. In parallel, we measured the statistical properties of the tidal potential reprojected back onto the boundary. The statistical one and two point expectations of the inflow were tabulated both kinematically and angularly on the  $R_{200}$  virtual sphere. All measurements were carried for 15000 haloes undergoing minor (as defined) mergers between redshift  $z=1$  and  $z=0$ . The two points correlations are carried both angularly and temporally for the flux densities and the tidal field. We also provided a method to re-generate realization of the field, via Equation (38) and (F6).

We briefly demonstrated how a perturbative description of the dynamics of halos can propagate the statistical properties of environments down to the statistical properties of the halo’s response. The description of the environment involved the projection of the potential and the source on a basis of functions. This basis allowed us to decouple the time evolution from the angular and velocity dependance of these two quantities. Hence, the accretion and the tidal potential were completely described by the projection coefficients and their statistical properties, which depend on time only. We also discussed how the flux densities of matter, momentum and energy can be related to the source and its expansion coefficients. We restricted ourselves to the one and two statistical description of the tidal field  $\psi^e$  and the flux density of mass  $\varpi_\rho$  and postponed to paper III (Aubert & Pichon (2006)), the full description of the higher moments. Since these measurements will be used as an entry to a perturbative description of the inner dynamics of haloes, only objects with quiet accretion history were selected as discussed in Section 4.2. Throughout this biased sample of haloes we made statistical measurements of the kinematic properties of accretion and derived results on the following quantities :

- the evolution of the accretion rate at the virial sphere: the net accretion is found to decrease with time, probing both the increasing contribution of outflows and the decline of strong interactions.
- the evolution of the net velocity distribution of the accretion : infall exhibits a typical velocity of  $0.75V_c$ . A backsplash component is detected at recent times with a significant outflowing component at a lower velocity ( $\sim 0.6V_c$ ).

- the evolution of the impact parameters/incidence angle distribution of the infall. The infall is found to be mainly radial while outflows are on more circular orbits.

- the angulo-temporal two-point correlation of the external potential on Virial sphere. The potential appear to be mainly dominated by a strong and constant quadrupole. The coherence time of smaller angular scales provide hints of an anisotropic accretion.

- the angular power spectrum of accreted matter. The clustering is dominated by small angular scales, possibly at the resolution limit.

- the angulo-temporal correlation of the flux density of mass. The coherence time appears shorter than for potential fluctuations, as expected.

These results can be interpreted in terms of properties of accreted objects or of smooth accretion and are coherent with previous studies (e.g. Mamon et al. (2004), Gill et al. (2004), van den Bosch (2002b), Knebe et al. (2004), Aubert et al. (2004)). This agreement between a fluid description of the environment and those published results are clearly encouraging for the reliability of the method. To this qualitative agreement, the current work adds some statistical significance and consequently allows quantitative constrains on the generic behaviour of dark matter accreted by haloes. A systematic comparison of the current approach with an analysis in terms of satellites is in order, in the spirit of Aubert et al. (2004).

These kinematic signatures provide insights on the processes which occur in the inner regions of haloes. In particular, the kinematic discrepancies between the different components of the mass flux should be understood in terms of dynamical friction, tidal stripping or even satellite-satellite interactions within the halo. The kinematical properties of accreted matter may be transposed to the kinematical properties of satellites observed around galaxies. Newly accreted material exhibits kinematic signatures (radial, high velocity trajectories) different from the ones measured for matter which already interacted with the halo (tangential, low-velocity orbits). Admittedly, it is not straightforward to apply directly these results to the luminous component (see paper I for a discussion of thresholding), and to see how projection effects may affect the distributions. Nevertheless, the corresponding observationnal measurements on satellites should provide information on the past history of these objects.

As discussed in paper I, these measurements can be used as an entry to the perturbative theory of the response of the open halo. Phenomena related to accretion can be consistently assessed via this framework: dynamical friction, tidal stripping and phase mixing. With the statistical description of the tidal field presented in this paper only, we may already implement the theory presented in paper I in the regime of pure tidal excitation. The complete knowledge of the source (which will be completed in paper III) should considerably extends the realm of application provided by this theory. Specifically, we have shown in paper I that the internal dynamics of sub structures within galactic haloes (distortion, clumps as traced by Xray emissivity, weak lensing, dark matter annihilation, tidal streams ..), and the implication for the disk (spiral structure, warp, etc...) could be predicted within this framework. Conversely the knowledge of the observed properties of a statistical sample of galactic halos could be used to (i) constrain observationnaly the statistical nature of the infall (ii) predict the observed distribution and correlations of upcoming surveys, (iii) time reverse the observed distribution of clumps, and finally (iv) weight the relative importance of the intrinsic (via the unperturbed distribution function) and

external (tidal and/ or infall) influence of the environment in determining the fate of galaxies.

The current measurements reduce the degree of freedom that still exists in the setting of numerical experiments in a galactic context. For instance, given that the structure of the external tidal field is found to be simple, it can easily be modelled as an external component in numerical simulations (or even in analytical studies). It would provide a simple but statistically relevant contribution of the large scale structures to the dynamical states of haloes. The temporal coherence of the first  $\ell > 2$  angular harmonics of the tidal field should allow one to draw more accurate representations of external contribution to the field that would include the fluctuations due to smaller structures. The kinematics of accretion is not random as well and the distribution of velocities at  $R_{200}$  follows a gaussian-shaped curve which characteristics evolve with time and exhibits a certain distribution of the impact parameter. These results put prescriptions that could be used to generate encounters between satellites and galactic disks that follow the ones measured in cosmological simulations at large radii. We also presented first constrains on the angular-temporal correlation function of the accretion. Even though it is not completely clear yet how resolution will eventually affect these results, such functions contains some glimpse of informations regarding the angular distribution of encounters with external systems but also regarding the frequency of accretion events. This frequency can also be probed by the temporal coherence of the fluctuations in the tidal field. The apparent contradiction that exists between the observed number of discs and the predicted large number of mergers may be solved by a better knowledge of the frequency of the latter: it may be low enough to solve this contradiction. In this context, simulations of successive mergers between a galaxy and satellites should be consistent with large scale simulations as we provide first constrains on the rate of minor encounters at the halo's outer boundary.

As argued in Pichon & Aubert (2006), we emphasize that an a priori discrimination between 'objects' and diffuse matter may not constitute the best way to describe accretion: it is not clear that luminous matter is always attached to dark matter overdensities, there is no unambiguous definition of substructures and their state change with time under the influence of tidal shocks or dynamical friction. The generation of objects that follow the current results is admittedly the easiest way to proceed but should be followed by a more general description of matter in terms of 'fluid approach', where 'objects' only constitute a specific phase of such a fluid. The statistical measurements on both  $\psi^e$  and  $\varpi_\rho$  allow the regeneration of synthetic environments. Knowing the average evolution and the angular power spectra of these quantities, the generation of spherical maps in the gaussian regime is straightforward. We describe such a regeneration procedure in appendix F. Such maps would efficiently provide realistic environments of halos, consistent (up to two-points statistics<sup>5</sup>) with those measured in cosmological simulations and could be 'embedded' into simulation of galaxies. Again, virialised structures would be naturally included (since they have their own statistical signature on the virial sphere) without relying on any adhoc prescription on their nature.

It should again be emphasized that some aspects of the present work are exploratory only, in that the resolution achieved ( $M_{\text{halo}} > 5 \cdot 10^{12} M_\odot$ ) is somewhat high for  $L_*$  galaxies. In fact, it will be

<sup>5</sup> Extensions to non gaussian fields are possible (following e.g. Contaldi & Magueijo (2001)) but imply the use of higher order correlations which are beyond the scope of this work.

interesting to confirm that the properties of infall do not asymptote for lower mass ( $M_{\text{halo}} < 5 \cdot 10^{12} M_\odot$ ) together with the intrinsic properties of galaxies. In addition a systematic study of biases induced by our estimators of angular correlations should be conducted. For a fixed halo mass, our lack of resolution implies that we over estimate the clumsiness of the infall.

As demonstrated in Section 6.2.2, the limited resolution (both spatially and in mass) of our simulations appeared to be an issue for some of the results presented here (e.g. the angular power spectrum of  $\varpi_\rho$ ). Systematic use of higher resolution simulations (in the spirit of section 6) will be required to fully assess these limitations. In particular, a fraction of the accretion detected as a weak/diffuse component may be associated to unresolved objects; the influence of small mass satellites should therefore be explored. With the prospect of deducing the properties of galaxy from halos environments, lower mass haloes are more likely to host only one galaxy, making them more suitable for such a study. Cosmological simulation of small volumes also tend to prevent the formation of rare events which may be relevant for the representativity of the study: for example, some disks seem to indicate that they were formed in 'very quiet' environments. The right balance between resolution and volume should be found. Aside from these biases induced by simulations, we also introduced selection criteria on both the mass or the accretion history of haloes and the influence of these arbitrary choices on our statistical distribution should be assessed precisely.

Eventually using hydrodynamical codes which include baryonic effects in simulations and introducing the physics of gas in our model, we would construct a complete semi-analytic tool to study the detailed inner dynamics of galaxies.

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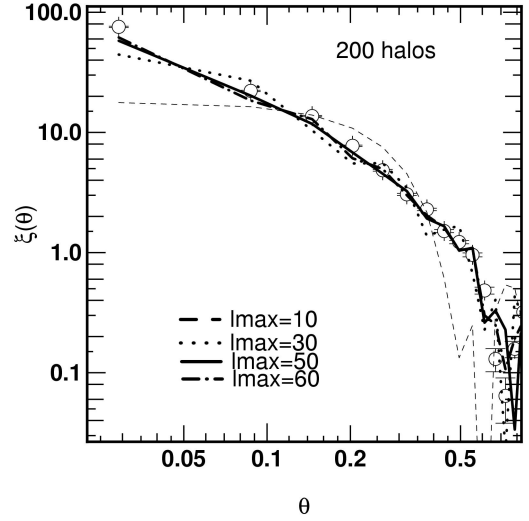
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## APPENDIX A: HARMONIC CONVERGENCE

As explained in section 3.1, the angular dependence of the source function is expanded over a basis of spherical harmonics  $Y_{\ell m}(\Omega)$ . In order to set the maximal order of this expansion,  $\ell_{\max}$ , we compared the two-point correlation function of the spherical field  $\rho_{v_r}(\Omega)$  to the one inferred from harmonic coefficients,  $a_{\ell m}$ , and power spectrum  $C_\ell$  (see equations (equation (42)) and (equation (43))). Within a set of randomly distributed particles, let



**Figure A1.** The average angular two-point correlation function,  $\langle \xi(\theta) \rangle$  of the advected mass spherical field  $\rho_{v_r}(\Omega)$ . The correlation function is shown as a function of the angular distance on the sphere  $\theta$  given in radians. Using the positions of accreted particles around 200 halos at  $z=1$ , the average correlation function can be computed (dots). Lines represent the correlation function deduced from the harmonic coefficients of the  $\rho_{v_r}(\Omega)$  fields around the same 200 halos, with  $\ell_{\max} = 10, 30, 50, 60$ . The convergence is ensured for  $\ell_{\max} \geq 50$ .

$d_{\text{poisson}}(\theta)$  be the probability of finding two particles with an angular separation  $\theta$ . If  $d(\theta)$  is the same probability for a given distribution of particles then its two points correlation function  $\xi(\theta)$  is defined as:

$$\xi(\theta) \equiv \frac{d(\theta)}{d_{\text{poisson}}(\theta)} - 1. \quad (\text{A1})$$

The correlation function  $\xi(\theta)$  and the  $a_{\ell m}$  coefficients are related by (e.g. Peacock (1999)) :

$$\xi(\theta) = \sum_{\ell=0}^{\ell_{\max}} C_\ell (2\ell + 1) P_\ell(\cos \theta), \quad (\text{A2})$$

where  $\theta$  is an angular distance on the sphere and  $P_\ell(x)$  a Legendre function. The average angular correlation function  $\langle \xi(\theta) \rangle_{\text{pair}}$  is defined as:

$$\langle \xi(\theta) \rangle_{\text{pair}} \equiv \frac{1}{\sum_p n_p^2} \sum_p n_p^2 \xi_p(\theta), \quad (\text{A3})$$

where  $\xi_p(\theta)$  is the two-point correlation function of the  $p$ -th halo computed using  $n_p$  particles passing through the virial sphere. From a set of 200 halos extracted from a simulation, we computed  $\langle \xi(\theta) \rangle$  using different values for  $\ell_{\max}$  (see Fig. A1). From the same set of halos, we also computed the average two point correlation function directly from the particles positions using equation (A1).

From Fig. A1, it clearly appears that  $\langle \xi(\theta) \rangle_{\text{pair}}$  has not converged for  $\ell_{\max} \leq 30$ . For  $\ell_{\max} \geq 50$  the actual two point correlation function is well reproduced. Since no real difference can be distinguished between  $\ell_{\max} = 50$  and  $\ell_{\max} = 60$ , we chose to limit the harmonic expansion of the source term to  $\ell \leq 50$ . Note that the truncation in  $\ell_{\max}$  defines an effective resolution beyond which the distribution is effectively coarsened.

## APPENDIX B: ANGULO-TEMPORAL CORRELATION

Let us consider a spherical field  $X(\Omega, t)$  which can be expanded over the spherical harmonic basis:

$$X(\Omega, t) = \sum_{\ell m} x_{\ell m}(t) Y_{\ell m}(\Omega). \quad (\text{B1})$$

The correlation  $w^X$  between two successive realizations of  $X$  is defined as:

$$w^X(\Omega, \Omega', t, t') \equiv \langle X(\Omega, t) X(\Omega', t') \rangle, \quad (\text{B2})$$

where  $\langle \cdot \rangle$  stands for the statistical average. If  $X(\Omega, t)$  is *isotropic*, the correlation should not depend on  $\Omega$  or  $\Omega'$  but only on the distance  $\theta$  between the two points. It implies that  $w^X$  can be expanded on a basis of Legendre Polynomials  $P_L(y)$ :

$$w^X(\Omega, \Omega', t, t') = w^X(\theta, t, t') = \sum_L (2L+1) T_L P_L(\cos(\theta)). \quad (\text{B3})$$

How are  $T_L$  and  $x_{\ell m}$  related? Rewriting equation (B2) as:

$$w^X(\theta, t, t') = \sum_{\ell m} \sum_{\ell' m'} \langle x_{\ell m}(t) x_{\ell' m'}^*(t') \rangle Y_{\ell m}(\Omega) Y_{\ell' m'}^*(\Omega'), \quad (\text{B4})$$

one can write:

$$\int d\Omega d\Omega' Y_{\ell_1 m_1}^*(\Omega) Y_{\ell_2 m_2}(\Omega') w^X = \langle x_{\ell_1 m_1}(t) x_{\ell_2 m_2}^*(t') \rangle. \quad (\text{B5})$$

Meanwhile, assuming isotropy, one can also write

$$\begin{aligned} & \int d\Omega d\Omega' Y_{\ell_1 m_1}^*(\Omega) Y_{\ell_2 m_2}(\Omega') w^X = \\ & \sum_L (2L+1) T_L \int d\Omega d\Omega' Y_{\ell_1 m_1}^*(\Omega) Y_{\ell_2 m_2}(\Omega') P_L(\cos(\theta)) \\ & = \sum_{LM} (4\pi) T_L \int d\Omega d\Omega' Y_{\ell_1 m_1}^*(\Omega) Y_{\ell_2 m_2}(\Omega') Y_{LM}(\Omega) Y_{LM}^*(\Omega') \\ & = \sum_{LM} (4\pi) T_L \delta_{L\ell_1} \delta_{L\ell_2} \delta_{Mm_1} \delta_{Mm_2}, \end{aligned} \quad (\text{B6})$$

where  $P_L$  is expressed in terms of spherical harmonics using the spherical harmonics addition theorem. In the end, we get:

$$T_\ell = \frac{1}{4\pi} \langle x_{\ell m}(t) x_{\ell m}^*(t') \rangle. \quad (\text{B7})$$

For a given realization of  $X(\Omega, t)$ ,  $T_\ell$  can be estimated by:

$$T_\ell = \frac{1}{4\pi} \frac{1}{2\ell+1} \sum_m x_{\ell m}(t) x_{\ell m}^*(t'). \quad (\text{B8})$$

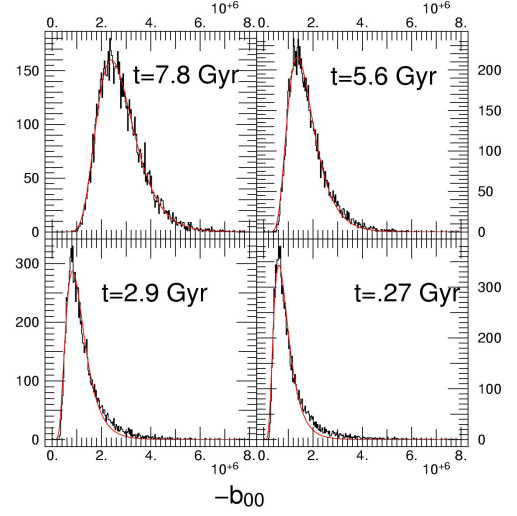
## APPENDIX C: DISTRIBUTIONS

In this appendix, we present the various distributions fitted either by normal or log-normal PDF. For each quantity, the mode (or most probable value) of the distribution has been obtained from these fits. The gaussian distribution is defined by:

$$N(x) = \frac{A}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \quad (\text{C1})$$

while the mode is equivalent to the mean  $\mu$ . The log-normal distribution is given by

$$\text{LN}(x) = \frac{A}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\log(x/\mu))^2}{2\sigma^2}\right), \quad (\text{C2})$$



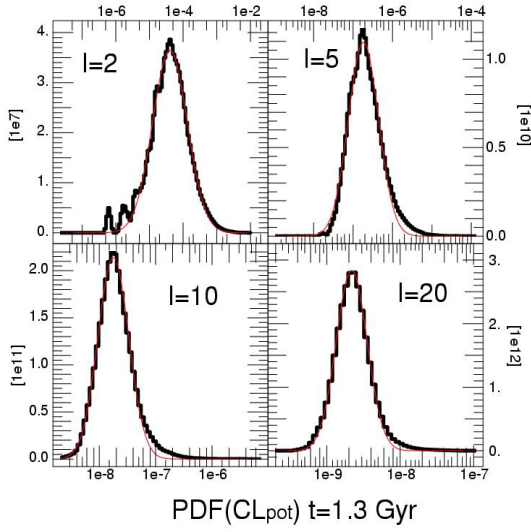
**Figure C1.** The probability distribution of  $b_{00}$  at four different times. This coefficient is proportional to the external potential averaged on the sphere. The log-normal fit is also shown.

while the mode is given by  $\mu \exp(-\sigma^2)$ . The different fits mentioned in the main text are described in the following figures:

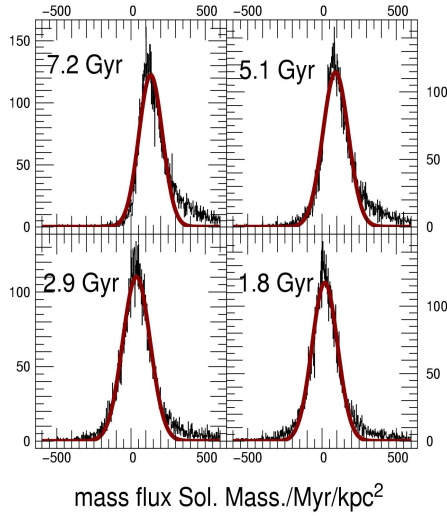
- Fig. (C1) shows the distributions of the harmonic coefficient  $b_{00}(t)$  which is proportional to the potential averaged on the sphere. It is expressed in units of  $GM/R$  where  $M$  is expressed in  $10^{10} M_\odot$ ,  $R$  in  $\text{kpc} h^{-1}$  and  $G=43007$  in internal units.
- Fig. (C2) shows the distributions of the external potential's power spectrum for four different harmonic  $\ell = 2, 5, 10, 20$  at  $t=1.3$  Gyr. The distribution has been fitted by a log normal function.
- Fig. (C3) shows the distributions of the mean flux  $\Phi^M(t)$ . The mean flux is proportional to the harmonic coefficient  $a_{00}$ . The distribution is fitted by a normal distribution. The normal model agrees well with the measured distribution at recent times but fail to reproduce the outliers tail at high redshift. Consequently the mode position is underestimated at these times.
- Fig. (C6) shows the distributions of one of the coefficient involved in the computation of the velocity distribution  $\phi(v)$ . Four different times are being represented. The coefficient distribution has been fitted by a gaussian distribution.
- Fig. (C4) shows the distributions of the power spectrum values  $C_\ell^{\omega\rho}$  for four different harmonic order  $\ell$ . The fits were made at  $t = 1.9$  Gyr. This distribution has been fitted by a lognormal distribution.
- Fig. (C5) shows the distributions of the power spectrum values  $C_\ell^{\omega\rho}$  for four different harmonic order  $\ell$ . The fits were made at  $t = 2.95$  Gyr. This distribution has been fitted by a lognormal distribution.

## APPENDIX D: FITS AND TABLES

In the main text, some statistics are fitted by simple laws and the time evolution of these statistics can be described by the time evolution of the fitting parameters. In this appendix, we present the time evolution of these parameters:



**Figure C2.** The probability distribution of  $C_\ell^{\psi_e}$  for  $\ell = 2, 5, 10, 20$  at the look-back time  $t = 1.3$  Gyr. Note that the X-axis is sampled logarithmically; the corresponding log normal fit of the mode is also shown.

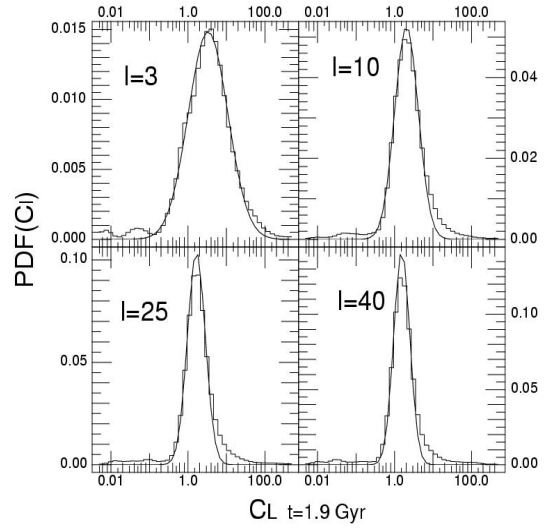


**Figure C3.** The probability distribution of the mean flux  $\Phi^M(t)$  for four different times. The normal fit is also shown.

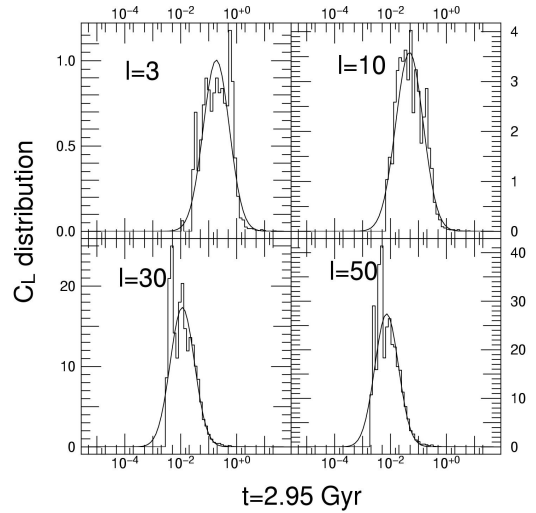
- Fig. (D1) and Table D1 presents the time evolution of the velocity distribution of accretion. It can be fitted by two gaussians drifting toward higher velocities with time.

- Table D2 summarize the time evolution of the distribution of incidence angles. This distribution can be fitted by a gaussian (for the inflows) and a linear relation (for the outflows). The fitting parameters present a linear evolution with time.

- Fig. (D2) and Table D3 summarize the time evolution of the angular power spectrum of  $\varpi_\rho$ . The power spectrum can be fitted by equation (76) where fitting parameters decreases exponentially with time.



**Figure C4.** The probability distribution of the  $C_\ell$  for  $\ell = 3, 10, 25, 40$  at the look-back time  $t = 1.9$  Gyr. Note that the X-axis is sampled logarithmically; the corresponding log normal fit of the mode is also shown.

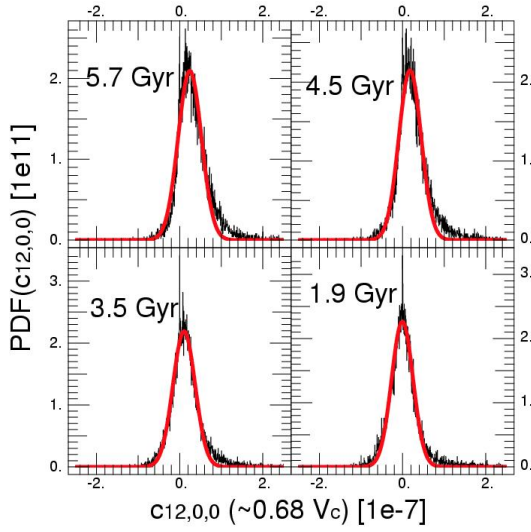


**Figure C5.** The probability distribution of  $C_\ell^{\sigma_\rho}$  for  $\ell = 3, 10, 25, 40$  at the look-back time  $t = 2.95$  Gyr. Note that the X-axis is sampled logarithmically; the corresponding log normal fit of the mode is also shown.

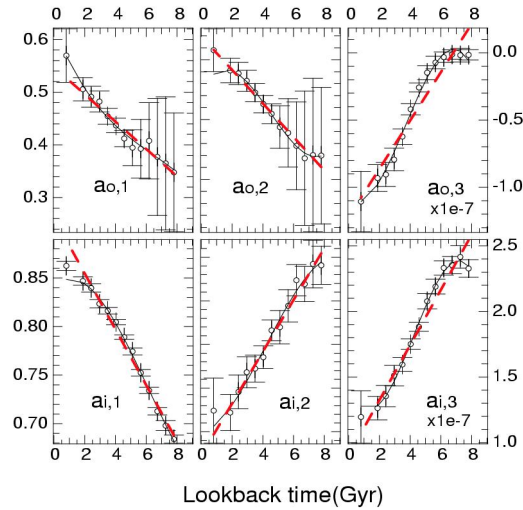
	$m$	$n$
$q_{i,1}$	-0.028	0.912
$q_{i,2}$	0.0059	0.084
$q_{i,3}$	$2.1 \cdot 10^{-8}$	$9.03 \cdot 10^{-8}$
$q_{o,1}$	-0.026	0.54
$q_{o,2}$	-0.028	0.23
$q_{o,3}^{(*)}$	$1.79 \cdot 10^{-8}$	$-1.22 \cdot 10^{-7}$

**Table D1.** Fitting parameters for the time evolution of Gaussian coefficients,  $q(t) = m \times t(\text{Gyr}) + n$ . These coefficients, together with equation (61) allow us to compute the time evolution of  $\langle \varphi(v, t) \rangle$ . (\*) By definition,  $q_{o,3} \leq 0$ .





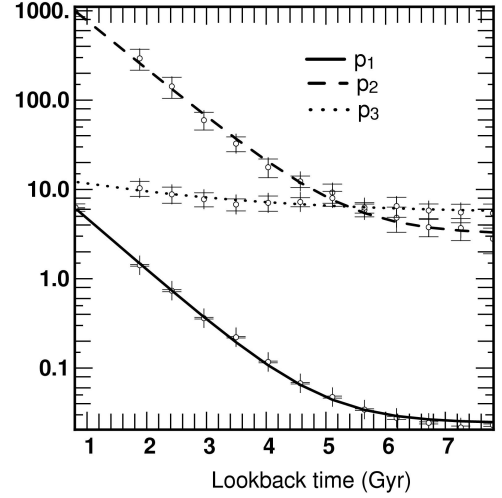
**Figure C6.** The probability distribution of the  $c_{12,0,0}^{0,0}$  for the look-back times  $t = 1.9, 3.5, 4.5$  and  $5.7$  Gyrs. Note that the X-axis is sampled linearly; the corresponding fit of the mode is also shown.



**Figure D1.** time evolution of the fitted coefficients (given in table D1) of the PDF  $\langle \varphi(v, t) \rangle$  for the parameterization suggested in equation (61) (symbols). Bars stand for the  $3\sigma$  errors on the coefficients determination.  $q_{i,k}, k = 1, 2, 3$  are respectively the mean, the r.m.s and the amplitude of the positive Gaussian (i.e. infalling component).  $q_{o,k}, k = 1, 2, 3$  are the corresponding values for the negative Gaussian (i.e. outflowing component). The trend is accurately fitted by a linear relation (red dashed line).

	$m$	$n$
$p_1$	-0.038	0.83
$p_2$	$-3.12 \cdot 10^{-8}$	$2.29 \cdot 10^{-7}$
$p_3$	$8.59 \cdot 10^{-8}$	$-6.64 \cdot 10^{-7}$

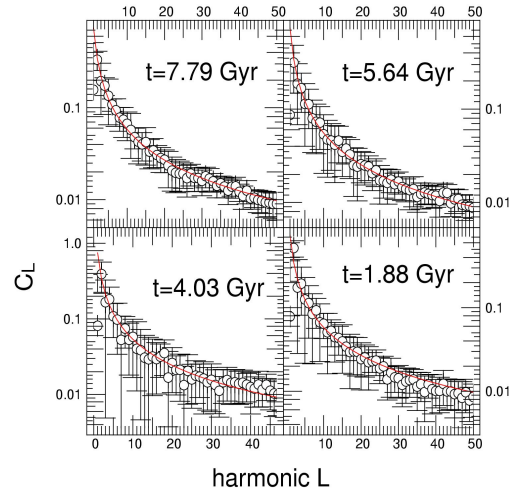
**Table D2.** Fitting parameters for the time evolution  $p_k(t), k = 1, 2, 3$  parameters.  $p(t) = m \times t(\text{Gyr}) + n$ . Together with equation (65), these coefficients allow to predict the time evolution of  $\langle \vartheta(\Gamma_1, t) \rangle$ .



**Figure D2.** Time evolution of the coefficients for the  $\langle C_\ell^{\varpi\rho} \rangle(t)$  model (see equation (76))(symbols). The three time evolutions may be described accurately by decreasing exponentials (see equation (77)) (solid lines).

	$h$	$k$	$u$
$q_1^{\varpi\rho}$	0.0242	17.86	0.867
$q_2^{\varpi\rho}$	3.153	2699.	0.8933
$q_3^{\varpi\rho}$	5.4105	9.44	1.55

**Table D3.** Fitting parameters for the time evolution of  $\langle C_\ell^{\varpi\rho} \rangle$  model's coefficients (see equation (76)).  $q_3^{\varpi\rho}(t) = h + ke^{-\frac{t}{u^2}}$  with lookback time  $t$  expressed in Gyr.



**Figure E1.** The angular power spectrum of the external potential,  $\langle C_\ell^{\varpi\rho} \rangle(t)$ , at four different lookback times (symbols). Harmonics coefficients were normalized using equation (E2), halo by halo.  $\langle C_\ell^{\varpi\rho} \rangle(t)$  is taken as the mode of the log-normal function used to fit the  $C_\ell^{\varpi\rho}$  distribution. Bars stand for  $3\sigma$  errors. For a given  $\ell$  the corresponding angular scale is  $\pi/\ell$ . The power spectra maybe fitted by a generic model given by equation (E3) (solid line). Unlike  $\langle C_\ell^{\varpi\rho} \rangle(t)$ ,  $\langle C_\ell^{\varpi\rho} \rangle(t)$  remains the same with time because of a different normalization.

### APPENDIX E: ALTERNATIVE CONTRAST AND ANGULO-TEMPORAL CORRELATION

It appears clearly from the time evolution of  $\langle\langle C_\ell^{\varpi\rho}\rangle\rangle(t)$  presented in the main text, that no angulo-temporal correlation can be computed, since it would be completely dominated by the secular evolution of the power spectrum. For this reason, an alternative definition of the flux density contrast has been used:

$$\delta'_{[\varpi\rho]}(\Omega) \equiv \frac{\varpi_\rho(\Omega) - \overline{\varpi}_\rho}{\overline{\varpi}_\rho} = \sum_{\ell,m} \tilde{a}'_{\ell,m} Y_{\ell,m}(\Omega), \quad (\text{E1})$$

or in terms of harmonic coefficients:

$$\tilde{a}'_{\ell,m} \equiv \sqrt{4\pi} \left( \frac{a_{\ell,m}}{a_{00}} - \delta_{\ell 0} \right). \quad (\text{E2})$$

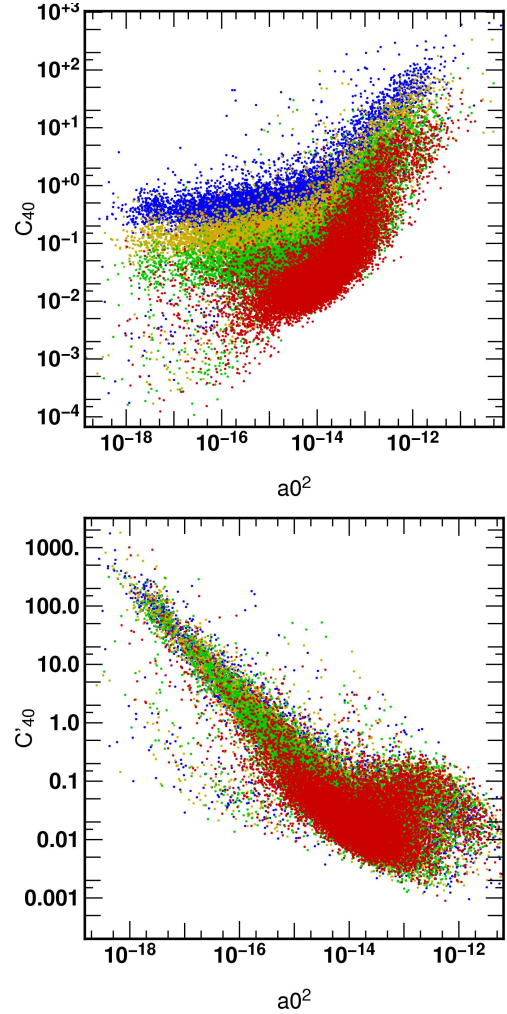
Using this new definition, the fluctuations of  $\varpi_\rho$  on the sphere are measured relative to the mass flux of each individual halo. The main drawback of such a definition is that it couples the  $a_{00}$  and the  $a_{\ell m}$  coefficients from the beginning. For example, the resimulation of such fields would require the knowledge of conditional probabilities, i.e. what is the distribution of  $a_{\ell m}$  for a given value of  $a_{00}$ , while the previous definition (given by equation (74)) only acted as specific choice of (non constant) units. Furthermore this new definition is more sensitive to outliers with low  $a_{00}$  values.

Still, the angular power spectrum  $\langle\langle C_\ell^{\varpi\rho}\rangle\rangle(t)$  of  $\delta'_{[\varpi\rho]}(\Omega)$  is much more regular than the one obtained from the previous definition. Its overall amplitude remains constant over the last 8 Gyrs, while its shape seems to be less dominated by small scale contributions. This alternative power spectrum is well fitted by a single power law :

$$\langle\langle C_\ell^{\varpi\rho}\rangle\rangle(t) = 0.75\ell^{-1.15}, \quad (\text{E3})$$

for the whole time range covered by the current measurements. This constant shape suggest that harmonic coefficients scale like  $a_{00}$ , i.e. the mass flux. Such a scaling is not obvious, since a strongly clustered  $\varpi_\rho$  field may coexist with a nil net flux (i.e.  $a_{00} \sim 0$ ). It also implies that the evolution measured on the previous definition of the power spectrum,  $\langle\langle C_\ell^{\varpi\rho}\rangle\rangle(t)$ , is more related to the evolution of the average flux (traced by  $a_{00}$ ) than to the modification of the fluctuations amplitude (traced by the others  $a_{\ell m}$ ). Still, the evolution of  $\langle a_{00} \rangle$  spans over one magnitude while the evolution of  $\langle\langle C_\ell^{\varpi\rho}\rangle\rangle(t)$  spans over several order of magnitudes: this strongly suggests that two different populations of haloes contribute to the two type of power spectrum. In Fig. (E2), the scatter plot of  $C_{40}$  and  $C'_{40}$  as a function of  $a_{00}$  shows that the haloes which experience strong accretion dominate the peak of the  $C'_{40}$  distribution while haloes with low accretion dominate the peak of the  $C_{40}$ . Furthermore  $C_{40}$  does not scale anymore like  $a_{00}$  as it drops below some level, providing hints of resolution and isolated particles effects. To conclude,  $\delta'_{[\varpi\rho]}(\Omega)$  appears as better way to rescale the fluctuations' amplitude since it provides a more regular behavior of the power spectrum, but it is a more complex quantity to manipulate. Meanwhile,  $\delta_{[\varpi\rho]}(\Omega)$  is probably the correct way to proceed but is clearly more sensitive to resolution effects, which should be assessed with bigger simulations in the future.

Since the behavior of  $\delta'_{[\varpi\rho]}(\Omega)$  is more regular than the previous contrast definition, the angulo-temporal correlation function of the flux density of mass has been computed from this new definition. Since this definition is different from that used for the potential, we restrict ourselves to a qualitative description. The correlations are given in Fig. (E3). Clearly, the correlation is more peaked around  $\Delta t = 0$  and more generally  $w^{\varpi\rho}$  is sharper than  $w^\psi$ . Note

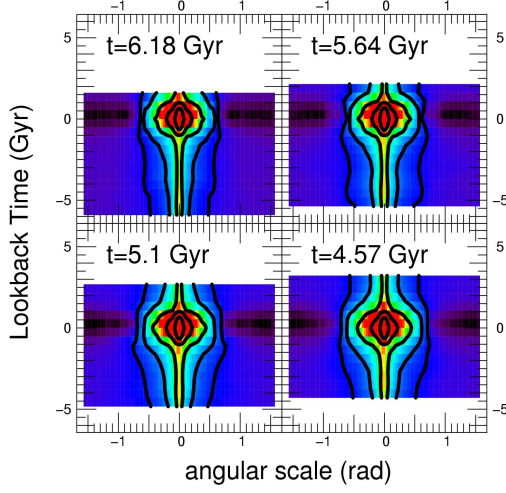


**Figure E2.** Scatter plots of the power spectra  $C_{40}^{\varpi\rho}$  (Top) and  $C'_{40}{}^{\varpi\rho}$  (Bottom) as a function of  $a_{00}^2$ . The four colors stand for different lookback times:  $t = 7.8$  Gyrs (red), 5.6 Gyrs (green), 4 Gyrs (yellow) and 2.8 Gyrs (blue). The monopole  $a_{00}$  scales like the integrated flux of matter. The quantities  $C_{40}^{\varpi\rho}$  and  $C'_{40}{}^{\varpi\rho}$  differ by the normalization applied to the harmonic coefficients  $a_{\ell m}$ . See the main text for more details.

that no multipole  $\ell$  has been removed during the computation of  $w^{\varpi\rho}$ , implying that the quadrupole effect measured in the potential correlation is simply not detected for this field. This strongly suggest a large-scale ‘cosmic’ origin for the quadrupolar tidal field rather than an artifact of the spherical intersection of an ellipse. Furthermore, the correlation time is smaller than the one measured for the potential, even compared to the correlation time of the potential without the  $\ell = 2$  component. This is coherent with the fact that density blobs should be ‘sharper’ than potential blobs as they pass through the sphere.

### APPENDIX F: RE-GENERATING GALACTIC TIME LINES

Given the measurements given in Section 5 and Section 6, let us describe here how to regenerate realizations of the history of the environment of haloes, first for the tidal field only, and then for the full accretion history.



**Figure E3.** The angulo-temporal correlation function,  $w^{\varpi}(\theta, \Delta t) = \langle \delta_{[\varpi]}(\Omega, t) \delta_{[\varpi]}(\Omega + \Delta\Omega, t + \Delta t) \rangle$ . Blue (resp. red) colors stand for low (resp. high) values of the correlation. Isocontours are also shown. Large angular scale isocontours ( $\Delta\Omega \sim \pi/2$ ) have large temporal extent, due to the quadrupole dominance over the potential seen on the virial sphere.

### F1 Re-generating tidal fields

Let us first focus on the generation of the potential tidal field generated by fly by's, hence neglecting the influence of the infall through the virial sphere.<sup>6</sup>

Let us call  $\hat{\Psi} = \hat{\psi}_{\mathbf{m},\omega}(T)$  the temporal (w.r.t the fast time) and angular Fourier transform of the potential, at fixed slow time,  $T$ . The probability distribution of  $\hat{\Psi}$ ,  $p(\hat{\Psi})$  is given by

$$p(\hat{\Psi}) = \frac{\exp\left[-\frac{1}{2}(\hat{\Psi} - \langle \hat{\Psi} \rangle)^{\top} \cdot \mathbf{C}_{\hat{\Psi}}^{-1} \cdot (\hat{\Psi} - \langle \hat{\Psi} \rangle)\right]}{(2\pi)^{1/2} \det^{1/2} |\mathbf{C}_{\hat{\Psi}}|}, \quad (\text{F1})$$

where the variance reads

$$\mathbf{C}_{\hat{\Psi}}(T) = \langle (\hat{\Psi}_{\mathbf{m},\omega} - \langle \hat{\Psi}_{\mathbf{m},\omega} \rangle) \cdot (\hat{\Psi}_{\mathbf{m},\omega} - \langle \hat{\Psi}_{\mathbf{m},\omega} \rangle) \rangle, \quad (\text{F2})$$

and the mean field obeys

$$\langle \hat{\Psi} \rangle(T) = \langle \hat{\Psi}_{\mathbf{m},\omega} \rangle. \quad (\text{F3})$$

Since the potential is isotropic,  $\langle \hat{\Psi} \rangle(T)$  is essentially zero (see also Fig. F1), while  $\mathbf{C}_{\hat{\Psi}}$  stands for the angular power spectrum described in section 6.1. Let us call  $\{\hat{\Psi}_{\mathbf{m},\omega}(T_i)\}_{i \leq N}$ , the set of sampled  $\hat{\Psi}$  a fixed slow time,  $T_i$ . Relying on a linear interpolation between two such realizations, the corresponding external potential reads in real space:

$$\psi^e(t, \Omega) = \sum_i \sum_{\mathbf{m}} \int d\omega \exp(i\mathbf{m} \cdot \Omega + i\omega t) \mathcal{K}_i^{\mathbf{m}}(t, \omega). \quad (\text{F4})$$

where the kernel,  $\mathcal{K}_i^{\mathbf{m}}(t, \omega)$ , reads

$$\mathcal{K}_i^{\mathbf{m}}(t, \omega) = \left[ \hat{\Psi}_{\mathbf{m},\omega}(T_{i+1}) \frac{t - T_i}{T_{i+1} - T_i} + \hat{\Psi}_{\mathbf{m},\omega}(T_i) \frac{T_{i+1} - t}{T_{i+1} - T_i} \right] \times \Theta(T_{i+1} - t) \Theta(t - T_i), \quad (\text{F5})$$

<sup>6</sup> note that this assumption is not coherent with the way the measurements are carried, since it implies that the infalling material somehow disappear after crossing  $R_{200}$ .

recalling that  $\Theta(x)$  is the Heaviside function. This procedure can be repeated for an arbitrary number of virtual tidal histories.

### F2 Re-generating tidal fields and infall history

Let us assume briefly that the fields are stationary both in time and angle, and that their statistics is Gaussian. As shown in Fig. F1, this assumption is essentially valid for the expansions of the potential and the flux density of mass. Let us call the eleven dimensional vector  $\mathbf{\Pi}(t) \equiv (\varpi_{\rho}, \varpi_{\rho\nu}, \varpi_{\rho\sigma\sigma}, \psi^e)$ ; and  $\hat{\mathbf{\Pi}}_{\mathbf{m},\omega}$  the temporal and angular Fourier transform of the fields. The joint probability of the field,  $p(\hat{\mathbf{\Pi}}_{\mathbf{m},\omega})$ , reads

$$p(\hat{\mathbf{\Pi}}_{\mathbf{m},\omega}) = \frac{\exp\left[-\frac{1}{2}(\hat{\mathbf{\Pi}} - \langle \hat{\mathbf{\Pi}} \rangle)^{\top} \cdot \mathbf{C}_{\hat{\mathbf{\Pi}}}^{-1} \cdot (\hat{\mathbf{\Pi}} - \langle \hat{\mathbf{\Pi}} \rangle)\right]}{(2\pi)^{11/2} \det^{1/2} |\mathbf{C}_{\hat{\mathbf{\Pi}}}|}, \quad (\text{F6})$$

where

$$\mathbf{C}_{\mathbf{\Pi}} = \left[ \langle (\hat{\mathbf{\Pi}}_{\mathbf{m},\omega}^i - \langle \hat{\mathbf{\Pi}}_{\mathbf{m},\omega}^i \rangle) \cdot (\hat{\mathbf{\Pi}}_{\mathbf{m},\omega}^j - \langle \hat{\mathbf{\Pi}}_{\mathbf{m},\omega}^j \rangle) \rangle \right]_{i,j \leq 11}, \quad (\text{F7})$$

and

$$\langle \hat{\mathbf{\Pi}} \rangle = \left[ \langle \hat{\mathbf{\Pi}}_{\mathbf{m},\omega}^i \rangle \right]_{i \leq 11}, \quad (\text{F8})$$

Since these fields are mostly isotropic, their expansions coefficients are nil on average. Hence, the quantity  $\mathbf{C}_{\mathbf{\Pi}}$  stands for the angular power spectrum of the eleven fields. For respectively the potential and the flux density of mass, its *temporal* evolution is described in section 6.1 and 6.2. These measured power spectra are sufficient to generate environments restricted to the flux density of mass and the potential. We emphasize that these two fields would not be coherent if no-cross correlations is specified. These cross-correlations and the power spectra for higher moments of the source are postponed to paper III.

Assuming the full knowledge of these 11 fields and their cross-correlation, it is therefore straightforward to generate for each  $(\mathbf{m}, \omega)$  a eleven dimensional vector which satisfies equation (F6), and repeat the draw for all modes (both  $\omega$  and  $\mathbf{m}$ ). Inverse Fourier transform yields  $\mathbf{\Pi}(t)$ . Once  $\mathbf{\Pi}(t)$  is known, we can regenerate the whole five dimensional phase-space source as a function of time via equation (38). This process can also be repeated for an arbitrary number of virtual halo histories. The assumption of stationarity in time can be lifted following the same route as that sketched in Section F1 (see Equation (F5)).

## APPENDIX G: FROM EXPANSION COEFFICIENTS TO FLUX DENSITIES

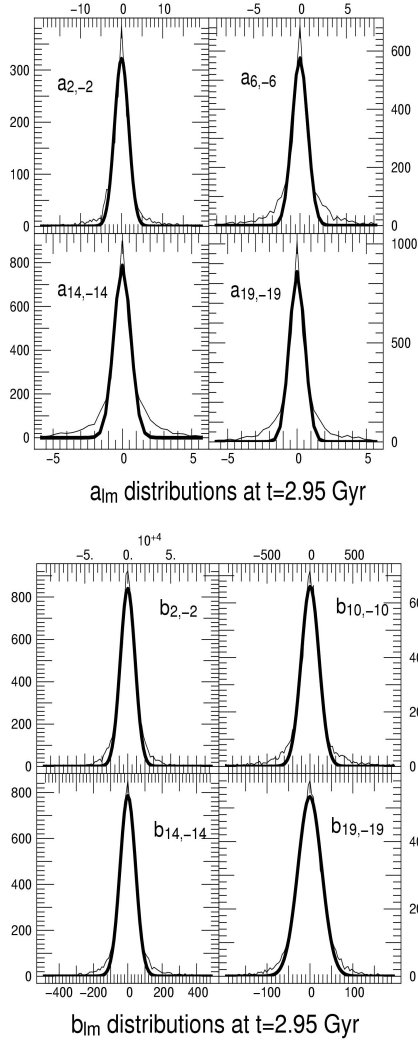
### G1 From expansion coefficients to advected momentum

The phase space distribution of advected momentum is given by:

$$\begin{aligned} \varpi_{\rho\nu}(\Omega, v, \Gamma, t) &\equiv s^e(\Omega, v, \Gamma, t) \mathbf{v}. & (\text{G1}) \\ &= \sum_{\alpha, \mathbf{m}, \mathbf{m}'} c_{\alpha\mathbf{m}\mathbf{m}'}^{\mathbf{m}}(t) g_{\alpha}(v) Y_{\mathbf{m}}(\Omega) Y_{\mathbf{m}'}(\Gamma) \mathbf{v}, & (\text{G2}) \end{aligned}$$

where the velocity vector may be written as a function of spherical harmonics:

$$\begin{aligned} \mathbf{v} &= -v \sqrt{\frac{2\pi}{3}} (-Y_{1-1}^*(\Gamma) + Y_{11}^*(\Gamma)) \mathbf{e}_{\theta} \\ &\quad -iv \sqrt{\frac{2\pi}{3}} (Y_{1-1}^*(\Gamma) + Y_{11}^*(\Gamma)) \mathbf{e}_{\phi} \end{aligned}$$



**Figure F1.** Probing the gaussianity of the harmonic expansions  $a_{\ell m}$ , describing the flux density of mass (*Top*) and  $b_{\ell m}$ , describing the potential  $\psi^e$  (*bottom*). Units are arbitrary. Only the real part of the coefficients is shown here, but the imaginary parts have similar distributions. Clearly, the distributions are quasi-gaussians.

$$v\sqrt{\frac{\pi}{3}}Y_{10}^*(\Gamma)\mathbf{e}_r. \quad (\text{G3})$$

Then, one can write:

$$\varpi_{\rho v}(\Omega, t) \equiv \int dv d\Gamma v^2 s^e(\Omega, v, \Gamma, t) \mathbf{v} \quad (\text{G4})$$

$$= \sum_{\alpha, m, m'} [\varpi_{\rho v}]_m(t) Y_m(\Omega) \quad (\text{G5})$$

where

$$[\varpi_{\rho v}]_m(t) = \sum_{\alpha} (3\sigma^2 \mu_{\alpha} + \mu_{\alpha}^3) \mathbf{T}_m(t), \quad (\text{G6})$$

and

$$\begin{aligned} \mathbf{T}_m(t) &= \sqrt{\frac{2\pi}{3}} (c_{\alpha,1,-1}^m(t) - c_{\alpha,1,1}^m(t)) \mathbf{e}_{\theta} \\ &+ i\sqrt{\frac{2\pi}{3}} (-c_{\alpha,1,-1}^m(t) - c_{\alpha,1,1}^m(t)) \mathbf{e}_{\phi} \end{aligned}$$

$$+ 2\sqrt{\frac{\pi}{3}} c_{\alpha,1,0}^m(t) \mathbf{e}_r. \quad (\text{G7})$$

## G2 From coefficients to advected velocity dispersion

The distribution of advected velocity dispersion is given by :

$$\varpi_{\rho\sigma_i\sigma_j}(\Omega, v, \Gamma, t) = s^e(\Omega, v, \Gamma, t) (\mathbf{v} - \mathbf{V}(\Omega, t))_i (\mathbf{v} - \mathbf{V}(\Omega, t))_j, \quad (\text{G8})$$

where the subscripts  $i$  and  $j$  stand for  $r, \theta, \phi$  and where

$$\mathbf{V}_i(\Omega, t) \equiv \frac{\int dv d\Gamma v^2 s^e(\Omega, v, \Gamma, t) v_i}{\int dv d\Gamma v^2 s^e(\Omega, v, \Gamma, t)} = \frac{\varpi_{\rho v_i}(\Omega, t)}{\varpi_{\rho}(\Omega, t)}. \quad (\text{G9})$$

Using equation (G8) and equation (G9), we find:

$$\varpi_{\rho\sigma_i\sigma_j}(\Omega, t) + \frac{\varpi_{\rho v_i}(\Omega, t) \varpi_{\rho v_j}(\Omega, t)}{\varpi_{\rho}(\Omega, t)} = \quad (\text{G10})$$

$$\begin{aligned} &\int dv d\Gamma v^2 s^e(\Omega, v, \Gamma, t) v_i v_j \\ &= \sum_{\mathbf{m}} [\mathbf{q}_{ij}(t)]_{\mathbf{m}} Y_{\mathbf{m}}(\Omega). \end{aligned} \quad (\text{G11})$$

The six independent elements of the symmetric tensor  $\mathbf{q}(t)$  can be computed from  $c_{\alpha m'}^m(t)$  coefficients using equation (G3) and recalling that:

$$\begin{aligned} \int d\Omega Y_{\ell_1, m_1} Y_{\ell_2, m_2} Y_{\ell_3, m_3} &= \sqrt{\frac{(2\ell_1 + 1)(2\ell_2 + 1)(2\ell_3 + 1)}{4\pi}} \times \\ &\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \end{aligned} \quad (\text{G12})$$

where  $\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = W_{m_1, m_2, m_3}^{\ell_1, \ell_2, \ell_3}$  is the Wigner 3-j symbol. One can find:

$$[\mathbf{q}_{rr}(t)]_{\mathbf{m}} = \sum_{\alpha m'} \mathcal{H}_{\alpha m'}^m(t) 2W_{000}^{11\ell'} W_{00m'}^{11\ell'} \quad (\text{G13})$$

$$[\mathbf{q}_{r\phi}(t)]_{\mathbf{m}} = \sum_{\alpha m'} \mathcal{H}_{\alpha m'}^m(t) i\sqrt{2} W_{000}^{11\ell'} (W_{-1-m'}^{11\ell'} + W_{11m'}^{11\ell'})$$

$$[\mathbf{q}_{r\theta}(t)]_{\mathbf{m}} = \sum_{\alpha m'} \mathcal{H}_{\alpha m'}^m(t) \sqrt{2} W_{000}^{11\ell'} (W_{-1-m'}^{11\ell'} - W_{11m'}^{11\ell'})$$

$$[\mathbf{q}_{\phi\phi}(t)]_{\mathbf{m}} = \sum_{\alpha m'} \mathcal{H}_{\alpha m'}^m(t) (-1) W_{000}^{11\ell'} (W_{11m'}^{11\ell'} + 2W_{-1-m'}^{11\ell'} + W_{-1-1m'}^{11\ell'})$$

$$[\mathbf{q}_{\phi\theta}(t)]_{\mathbf{m}} = \sum_{\alpha m'} \mathcal{H}_{\alpha m'}^m(t) i W_{000}^{11\ell'} (W_{-1-m'}^{11\ell'} - W_{11m'}^{11\ell'})$$

$$[\mathbf{q}_{\theta\theta}(t)]_{\mathbf{m}} = \sum_{\alpha m'} \mathcal{H}_{\alpha m'}^m(t) W_{000}^{11\ell'} (W_{11m'}^{11\ell'} - W_{-1-1m'}^{11\ell'} - 2W_{-1-m'}^{11\ell'}).$$

$$\text{where } \mathcal{H}_{\alpha m'}^m(t) = \sqrt{4\pi(2\ell' + 1)} (6\sigma^2 \mu_{\alpha}^2 + \mu_{\alpha}^4) c_{\alpha m'}^m(t)$$